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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 8: dividing a polynomial by a monomial and by a polynomial, factoring, identifying common factors, multiplying sum and difference of two numbers, factoring differences of two squares, and squaring a binomial. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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VOLUME

8

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

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P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME 8

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 8

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IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
1	1	$\frac{1}{1}$
1	2	$\frac{30}{2}$
1	3	$\frac{41}{2}$
1	4	$\frac{83}{1}$
1	5	$\frac{118}{1}$

VOLUME 8

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Dividing polynomial by monomial	6 - 11	8 - 6	8 - 5
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3	Factoring in algebra	7 - 1	11- 1	
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	Factoring difference of two squares	7 - 4	11- 5	8 - 6
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READING ASSIGNMENT

VOLUME 8

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	219	221
2	221	223
3	237	244
4	245	248
5	248	251

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co. 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

VOLUME NO. 8

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	220	1 , 2 , 7	08110
2	220	8 , 11 , 13	08110
3	220	15 , 18 , 19 , 20	08110
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5	223	2 , 4	08210
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7	223	17 , 19 , 21	08210
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9	239	15 , 17 , 18 , 20	08310
10	240	1 , 2 , 4 , 6 , 8	08310
11	242/2	3 , 11 , 18 , 21	08320
12	243	23 , 25 , 30	08320
13	246/1	16 , 19 , 20 , 23	08410
14	246/2	4 , 9 , 17 , 19	08410
15	247/2	2 , 5 , 9	08420
16	247/2	15 , 17 , 20	08420
17	249	3 , 4 , 5 , 7	08510
18	249	9 , 10 , 12 , 14 , 16	08510
19	250	17 , 20 , 22 , 24	08510
20	250	25 , 26 , 27 , 28	08510

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

$\frac{1}{1}$

Volume 8 Segment 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and 50	3 6	(Sequence Number)
	54 and 56	0 4	(Type of Punch Card)
	60 and 62	0 8	(Volume Number)
	66 and 68	0 1	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 219 - 221
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize which one of the following expressions is the same as

$$\frac{42 + 9}{3}$$

and choose the letter next to the correct answer.

(A) $39 + 6$

(B) $\frac{42}{3} + \frac{9}{3}$

(C) $13 + 3$

(D) None of these.

$\frac{1}{2}$

You made a wrong choice. You did not divide each of the terms in the numerator by the denominator. Consider, for example, the following quotient

$$\frac{39p - q}{q}$$

By the distributive property, $39p$ should be divided by q,

and $-q$ should also be divided by q.

Please return to page $\frac{27}{1}$ and try this question again.

$\frac{2}{1}$

Very good. You made the right choice. Here is how this question can be answered:

$$\frac{15x + 25y}{5}$$

is the same as,

$$\left(\frac{1}{5}\right)(15x + 25y)$$

this equals,

$$\left(\frac{1}{5}\right)(15x) + \left(\frac{1}{5}\right)(25y) \quad \text{by the distributive property.}$$

which equals:

$$3x + 5y$$

Please proceed to question 5 below.

$\frac{2}{2}$

Question 5

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{12x^3 + 6x}{3x} \quad \text{equals:}$$

(A) $4x^3 + 2x$

(B) $9x^2 + 3$

(C) $6x^2$

(D) $4x^2 + 2$

3/1

Very good. You made the correct choice. Here is how the quotient

$$\frac{8m - n}{n}$$

can be simplified: We re-write the quotient as $\left(\frac{1}{n}\right)(8m - n)$. Then applying the distributive property, we get

$$\frac{8m}{n} - \frac{n}{n}$$

Now, $\frac{8m}{n}$ cannot be simplified any further, since the numerator and the denominator consist of different letters. The second term is a number divided by itself, with a negative sign in front of it. Therefore, it equals -1. Hence, the correct answer is

$$\frac{8m}{n} - 1.$$

Please proceed to question 7 below.

3/2

Question 7

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{8a^3 - 4a}{2a}$$

equals:

(A) $4a^2 - 2$

(B) $6a^2 - 2$

(C) $4a^2 - 2a$

(D) $2a^2$

$\frac{4}{1}$

To simplify a numerical expression of the type:

$$\frac{75 - 45 + 15}{5}$$

instead of combining the terms of the numerator and then dividing we can use the distributive property. The above expression can be re-written as:

$$\left(\frac{1}{5}\right)(75 - 45 + 15) = \left(\frac{1}{5}\right)(75) - \left(\frac{1}{5}\right)(45) + \left(\frac{1}{5}\right)(15)$$

$$= 15 - 9 + 3$$

$$+ 9$$

It is important to observe the signs of the terms in the numerator.

Return to page $\frac{23}{1}$ and try this question again.

$\frac{4}{2}$

We don't agree. Let us try to make this business of dividing a polynomial by a number a little more real to you. Suppose you were told that a certain child was 4 years, 8 months, 2 weeks and 6 days old, and that a second child was half the age of the first child. How could you find the age of the second child? To obtain the age of the second child, you would divide the number of years by 2, the number of months by 2, the number of weeks by 2 and the number of days by 2. Well, you have actually followed the rule for dividing a polynomial by a number. The age of the first child <u>is a polynomial</u>; namely;

$4y + 3m + 2w + 6d$, where the letters stand for years, months, weeks, and days.

Similarly, in question 8, you should divide each term of the numerator by 3.

Return to page $\frac{17}{2}$ and try this question again.

$\frac{5}{1}$

You subtracted the numerical coefficients when you should have divided them. Let us try to make this point clear to you.

Consider for example: $\frac{27y^3}{3y}$

This quotient can be written as $\frac{27}{3} \cdot \frac{y^3}{y}$

Now, the numerical coefficient simply is 27 divided by 3, and that equals 9. The expression $\frac{y^3}{y}$ is the quotient of 2 powers with the same base. Only to this quotient do we apply the rule for the quotient of powers.

Please return to page $\frac{2}{2}$ and choose another letter.

$\frac{5}{2}$

You did not make the right choice. The mistake you made is that you subtracted the numerical coefficients. The question can be broken up into the quotient of 3 monomials, thus;

$$\frac{24b^4 - 12b^3 + 15b^2}{3b^2} = \frac{24b^4}{3b^2} - \frac{12b^3}{3b^2} + \frac{15b^2}{3b^2}$$

The numerical coefficients of each of these terms, namely;

$$\frac{24}{3} - \frac{12}{3}, \text{ and } \frac{15}{3}$$

represent division of numbers.

Please return to page $\frac{11}{2}$ and choose another letter.

$\frac{6}{1}$

You should have remembered an old rule which states that only like terms can be combined. The terms $8a^3$ and $-4a$ are not like terms. The quotient of a binomial and a monomial can be broken up into 2 quotients of monomials. Thus,

$$\frac{8a^3 - 4a}{2a} - \frac{8a^3}{2a} - \frac{4a}{2a}$$

Now, each of these quotients can be simplified. Please refer to your notebook and study the rules for simplifying the quotient of 2 monomials

Return to page $\frac{3}{2}$ and try this question again.

$\frac{6}{2}$

Almost right, but in mathematics, almost right means wrong. We are going to let you find your own mistake without our help. All that we will ask you is to recall the rules for the quotient of signed numbers.

Please return to page $\frac{8}{1}$ and try this question again.

$\frac{7}{1}$

This choice is correct.

Now proceed to question 4 which follows:

Question 4

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{15x + 25y}{5} \text{ equals:}$$

(A) $3x + 5y$

(B) $10x + 20y$

(C) $5x + 3y$

(D) $3x + 20y$

$\frac{7}{2}$

You made a wrong choice. A number divided by itself is not equal to 0.

$$\frac{a}{a} = 1 \quad \text{since} \quad a = 1\,a$$

Thus, a number divided by itself equals 1.

Please return to page $\frac{18}{2}$ and try this question again.

$\frac{8}{1}$

This choice is correct.

Now proceed to question 10 which follows:

Question 10

Apply the proper principle to simplify the quotient.

$$\frac{12x^3y^3 - 6x^2y^2 + 18xy}{-6xy}$$

and select the letter which labels the correct answer.

(A) $-2x^2y^2 - xy - 3$

(B) $6x^2y^2 - xy - 3$

(C) $-2x^3y^3 + x^2y^2 - 3$

(D) $-2x^2y^2 + xy - 3$

$\frac{8}{2}$

Suppose we first simplify this quotient:

$$\frac{8x^3 - 4x^2 - 2x}{2x} = \frac{8x^3}{2x} - \frac{4x^2}{2x} - \frac{2x}{2x}$$

$$= 4x^2 - 2x - 1$$

Now, we can substitute 2 for x and evaluate the expression.

Please continue from here.

Return to page $\frac{28}{2}$ and try this question again.

$\frac{9}{1}$

You did not make the correct choice. Let us help you get started.

$$\frac{x + y}{y} = 3$$

$$\frac{x}{y} + \frac{y}{y} = 3$$

Note that a number divided by itself equals 1.

We are sure that you can continue from here on your own.

Please return to page $\frac{19}{2}$ and consider question 13 again.

$\frac{9}{2}$

Remember this formula:

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} + \frac{\text{REMAINDER}}{\text{DIVISOR}}$$

and it will help you decide which is the correct answer.

Each word used in mathematics (with the exception of some simple words) has a definite definition. It is dangerous to assume that you know what a word means unless you have already learned its official definition.

Please return to page $\frac{30}{2}$ and choose another letter.

$\frac{10}{1}$

It is true that the coefficients are in a decreasing order. However, you were asked to choose the polynomial whose variable was in a decreasing order.

Return to $\frac{25}{2}$ and try question 3 again.

$\frac{10}{2}$

You merely re-wrote the quotient in another way. As amatter of fact, you will come across three different ways of writing a problem in division; namely,

$$\frac{745}{15}, \quad 15\overline{)745} \quad , \quad 745 \div 15$$

The above three expressions all indicate the same operation. We will give you a start to help you make the right choice. Think of the decimal representation of integers. Thus, for example:

$$864 = 8(100) + 6(10) + 4$$

Please continue from here.

Return to page $\frac{26}{2}$ and try this question again.

Very good. You made the right choice. Here is how this quotient can be simplified:

$$\frac{18x - 9y + 6z - 3w}{3} = \frac{18x}{3} - \frac{9y}{3} + \frac{6z}{3} - \frac{3w}{3}$$ Now,

$$\frac{18x}{3} = 6x$$

$$-\frac{9y}{3} = -3y$$

$$\frac{6z}{3} = 2z$$

$$-\frac{3w}{3} = -w$$

Thus, the correct answer is:

$$= 6x - 3y + 2z - w$$

Please proceed to question 9 below

Question 9

Apply the proper principle to simplify the quotient:

$$\frac{24b^4 - 12b^3 + 15b^2}{3b^2}$$

and select the letter which labels the correct answer.

(A) $21b^2 - 9b + 12$

(B) $8b - 4b^{\frac{3}{2}} + 5$

(C) $8b^2 - 4b + 5$

(D) $27b^2$

$\frac{12}{1}$

To simplify a numerical expression of the type:

$$\frac{36 \quad + \quad 18 \quad + \quad 6}{3}$$

instead of adding first and then dividing, as an alternate method, we can use the distributive property. The above expression can be re-written as

$$\frac{1}{3}\,(\,36 \quad + \quad 18 \quad + \quad 6\,) \quad \text{which equals}$$

$$\frac{1}{3}\,(\,36\,) \; + \frac{1}{3}(\,18\,) \; + \frac{1}{3}(\,6\,) \; =$$

$$12 \quad + \quad 6 \quad + \quad 2 \quad = \quad 20$$

Please return to page $\frac{1}{1}$ and try this question again.

$\frac{12}{2}$

We don't agree.

You made two mistakes. Let us try this question together.

$$\frac{12x^3y^3 \quad - \quad 6x^2y^2 \quad + \quad 18xy}{-6xy}$$

can be separated into 3 quotients:

$$\frac{12x^3y^3}{-6xy} \qquad \frac{-6x^2y^2}{-6xy} \quad \text{and} \quad \frac{18xy}{-6xy}$$

Now, ask yourself:

What is the value of $\frac{12}{-6}$?

What is the value of $\frac{-6}{-6}$?

Return to page $\frac{8}{1}$ and try this question again.

$\frac{13}{1}$

The expression $\frac{15x + 25y}{5}$ signifies that each term of the numerator is divided by the denominator. Thus, the expression can be re-written as

$$\frac{15x}{5} + \frac{25y}{5}$$

We are sure that you can continue from here.

Return to page $\frac{7}{1}$ and choose another letter.

$\frac{13}{2}$

We do not agree. Let us separate this quotient into three quotients.

Thus: $$\frac{8a^3b}{4a^2b} - \frac{16a^2b}{4a^2b} + \frac{4a^2b}{4a^2b} .$$

Let us take the first quotient and work our way from left to right.

Thus, we have:

$$\frac{8}{4} \qquad \frac{a^3}{a^2} \qquad \frac{b}{b}$$

Now: $$\frac{8}{4} = 2 \qquad \frac{a^3}{a^2} = a \qquad \frac{b}{b} = 1 .$$

this becomes:

$$2a$$

Proceed in the same way with the other 2 quotients.

Return to page $\frac{18}{2}$ and try the question again.

VIII

$\frac{14}{1}$

Exponents are subtracted only when we have the quotient of the same letters raised to powers.

For example:

$$\frac{y^5}{y^2} = y^{5-2}$$

$$= y^3$$

However, a numerical quotient such as $\frac{18}{3}$ is merely the division of 18 by 3, and the answer is 6 and <u>not 15</u>. Now, examine your choice, and you will see that you subtracted the numerical coefficients when you should have divided them.

Return to page $\frac{3}{2}$ and try this question again.

$\frac{14}{2}$

You did not choose the right answer. It is always a good idea to check a problem in division by multiplying the quotient by the divisor and then adding the remainder. This should be equal to the dividend. Look at this example.

```
        245
25 ) 6138
     -50
     ----
      113
     -100
     ----
       138
      -125
      ----
        13  remainder
```

Now, we multiply the quotient 245 by the divisor 25

```
  245
   25
 ----
 1225
 490
----
6125
```

Finally, we add the remainder to the above product; thus, 6125 + 13

This sum must equal the divident, 6138

Return to page $\frac{20}{2}$ and try this question again.

This choice is correct.

Now proceed to question 3 which follows:

Question 3

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{ab^2 + ab + a}{a} \text{ equals ?}$$

(A) $b^2 + b$

(B) $ab^2 + ab + a$

(C) $b^2 + b + 1$

(D) $ab^2 + ab$

By missing terms we mean the missing powers of the variable, counting from the highest power down to the lowest power. For example, consider the polynomial,

$$x^7 - 3x^5 + x^2 - 3$$

Note that the highest power of x is 7. Therefore, we start from 7 and count down to 0. Thus, we get 7 , 6 , 5 , 4 , 3 , 2 , 1 , 0. Now, the polynomial does not have x raised to the powers 6 , 4 , 3 , 1. Hence, the missing terms are x^6 , x^4 , x^3 , x. Note, 3 can be considered $3x^0$.

Please return to page 43/2 and choose another letter.

$\frac{16}{1}$

You did not make the right choice. When you have to divide a polynomial by a monomial, it is a good idea to break up the expression into quotients of monomials. Let us illustrate this. Suppose that we have to simplify the quotient,

$$\frac{33a^4 + 22a^2}{11a}$$

The above can be written as the sum of the quotient of 2 monomials

$$\frac{33a^4}{11a} + \frac{22a^2}{11a}$$

Now, the numerical coefficients are divided, and the exponents of the letter are subtracted.

Return to page $\frac{2}{2}$ and try this question again.

$\frac{16}{2}$

We don't agree. The rule for finding the quotient of powers is to retain the base and <u>subtract</u> the exponents.

i.e. $\boxed{\frac{x^a}{x^b} = x^{a-b}}$ when $a > b$

Please return to page $\frac{11}{2}$ and try this question again.

Very good. Here is how this question can be answered:

$$\frac{8a^3 - 4a}{2a} = \frac{1}{2a}(8a^3 - 4a) \qquad \text{(By the Distributive Law)}$$

$$= \frac{8a^3}{2a} - \frac{4a}{2a}$$

$$= 4a^2 - 2$$

Note: $\frac{8}{2} = 4$

$\frac{a^3}{a} = a^2$

$\frac{-4}{2} = -2$

$\frac{a}{a} = 1$

Now, proceed to question 8 below.

Question 8

Apply the proper principle to simplify the quotient,

$$\frac{18x - 9y + 6z - 3w}{3}$$

and select the letter next to the correct answer.

(A) $6x - 3y + 2z - 3w$

(B) $6x - 3y + 2z - w$

(C) $12x - 3y + 2z - w$

(D) $15x - 6y + 3z - 6w$

18
1

The expression $\frac{ab^2 + ab + a}{a}$ can be re-written as:

$$\frac{1}{a}(ab^2 + ab + a) = \frac{ab^2}{a} + \frac{ab}{a} + \frac{a}{a}$$

Each of the three terms is a quotient of powers. If you refer to your notebook, you will find the rule for finding the quotient of powers.

Return to page 15/1 and try this question again.

18
2

This choice is correct.

Now, proceed to question 11 which follows.

Question 11

Apply the proper principle to simplify the quotient,

$$\frac{8a^3b - 16a^2b^2 + 4a^2b}{4a^2b}$$

and select the letter which labels the correct answer.

(A) $2a - 4b + 1$

(B) $2a - 4b$

(C) $2ab - 4b^2 + b$

(D) $2a^{\frac{3}{2}} - 4ab^2 + ab$

$\frac{19}{1}$

Very good. You made the right choice. One way to do this problem is to simplify the quotient first. Thus,

$$\frac{8x^3 - 4x^2 - 2x}{} = \frac{8x^3}{2x} - \frac{4x^2}{2x} - \frac{2x}{2x}$$

$$= 4x^2 - 2x - 1$$

Now, when x = 2, we have

$$= 4(2^2) - 2(2) - 1$$

$$= 16 - 4 - 1$$

$$= 11$$

It is interesting to note that the above procedure can be used to <u>check</u> your work. Let us explain. To check whether the quotient $\frac{8x^3 - 4x^2 - 2x}{2x} = 4x^2 - 2x - 1$, we evaluate the quotient for any numerical value of "x", and we evaluate the answer for the same value of "x". These two answers should be the same. That is,

$\frac{8(2^3) - 4(2^2) - 2(2)}{2(2)}$ = should also equal 11.

If we complete the calculations, we find out that the above does equal 11.

Please proceed to question 13 below.

$\frac{19}{2}$

<u>Question 13</u>

Apply the proper principle to find the ratio x:y, if:

$$\frac{x + y}{y} = 3$$

Select the letter next to the right answer.

(A) 1:2 (B) 2:1 (C) 1:3 (D) 3:1

$\frac{20}{1}$

You made the correct choice. It will be helpful in remembering these words if we write the formula,

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} + \frac{\text{REMAINDER}}{\text{DIVISOR}}$$

Thus, in this question, we have:

$$\frac{645}{23} = 28 + \frac{1}{23}$$

so that 645 is the dividend, 23 is the divisor, 28 is the quotient, and 1 is the remainder.

Please go on to question 2 below.

$\frac{20}{2}$

Question 2:

Perform the indicated operation and select the letter which labels the correct answer, $3487 \div 43$

(A) $78\frac{3}{43}$

(B) 79

(C) 81

(D) $81\frac{4}{43}$

The variable is in an ascending order from left to right. True, it is in descending order the other way, but the "left to right" order is the one that is always meant whether or not it is stated.

Return to page 25/2 and try question 3 again.

We are dealing with integers that are represented in the decimal system. In this system, position is important. Consider the integer 372. This number is not the same as 3 + 7 + 2, because 372 means 300 + 70 + 2. Let us try another number; say, 4378.

$$4378 = 4000 + 300 + 70 + 8$$

Please return to page 26/2 and try this question again.

$\frac{22}{1}$

You did not divide correctly. Maybe you should go back to the text assignment and study the division of a polynomial by a polynomial on Page 222.

If, $x + 5$ were the correct answer, then

$(x + 5)(x - 2)$ should equal

$x^2 - 7x + 10$ Check the multiplication.

You will see that you get

$x^2 + 3x - 10$

Return to page $\frac{37}{1}$ and try this question again.

$\frac{22}{2}$

Question 7:

Apply the proper principle to find the quotient,

$$(3x^2 + 7x - 6) - (3 + x)$$

and select the letter which labels the correct answer.

(A) $3x + 16$

(B) $3x + 16$ and remainder 42

(C) $3x - 2$

(D) $3x - 2$ and remainder $\frac{-12}{x + 3}$

This choice is correct.

Now proceed to question 2 which follows.

Question 2

Recognize which one of the following expressions is the same as

$$\frac{48 - 16}{16}$$

and choose the letter next to the correct answer.

(A) 3

(B) 31

(C) 2

(D) None of these.

An old mathematics rule is true no matter when you learned it. We are referring to the rule for finding the quotient of powers. The rule is: to find the quotient of powers having the same base, we keep the base and subtract the exponent of the denominator from the exponent of the numerator. Thus,

$$\frac{b^7}{b^3} = b^{7-3} = b^4$$

$$\frac{c^5}{c} = c^{5-1} = c^4$$

Please return to page $\frac{8}{1}$ and try this question again.

$\frac{24}{1}$

The rule for finding the quotient of powers having the same base is to retain the base and subtract the exponents. For example,

$$\frac{21a^3}{3a} = 7a^2$$

$$\frac{18b}{3b} = 6$$

Note that a letter written without an exponent means that the letter has the exponent 1.

Please return to page $\frac{2}{2}$ and try this question again.

$\frac{24}{2}$

You divided the exponents; you should have subtracted them. Please keep in mind the rule for finding the quotient of powers:

$$\boxed{\frac{x^a}{x^b} = x^{a-b}} \quad \text{where } a > b$$

Return to page $\frac{18}{2}$ and try this question again.

If you re-write the quotient as the difference of the quotient of two monomials, you will get,

$$\frac{8a^3 - 4a}{2a} = \frac{8a^3}{2a} - \frac{4a}{2a}$$

Now, the second quotient is <u>not</u> equal to 2a. Give this question a little more thought.

Please return to page $\frac{3}{2}$ and choose another letter.

This choice is correct.

Now proceed to <u>question 3</u> which follows.

<u>Question 3</u>

Recognize which polynomial is written in decreasing order of the variable x , and select the letter next to such a polynomial.

(A) $6x^4 - 5x^3 + 4x^5 - 3$

(B) $5x^2 - x + 10$

(C) $7x^2 - 5x^3 + 2x^4$

(D) $4 + 3x^2 + 2x$

$\frac{26}{1}$

We don't agree.

If you divided 5 among 5 people; how much money would each person get? That's easy, you say. Each person will get $1. If you divided 8 pieces of gum among 8 children; how many pieces would each child get? Again, the answer is 1. In other words, a number divided by itself is equal to 1.

Coming back to our question, we have

$$\frac{ab^2 + ab + a}{a} = \left(\frac{1}{a}\right)\left(ab^2 + ab + a\right) =$$

$$\frac{ab^2}{a} + \frac{ab}{a} + \frac{a}{a}$$

Observe that the last term is a number divided by itself.

Return to page $\frac{15}{1}$ and choose another letter.

$\frac{26}{2}$

This choice is correct.

Now proceed to question 5 which follows.

Question 5

Recognize which of the following represents the quotient $\frac{693x}{3}$ re-written as the quotient of a polynomial of three terms and a monomial and select the letter which labels the correct statement.

(A) $3\overline{)\,693}$

(B) $\frac{6(100)x + 9(10)x + 3x}{3}$

(C) $\frac{6x}{3} + \frac{9x}{3} + \frac{3x}{3}$

(D) $\frac{3(231x)}{3}$

$\frac{27}{1}$

This choice is correct.

Now proceed to question 6 which follows:

Question 6

Apply the proper principle to simpligy the quotient and select the letter which labels t he correct answer.

$\frac{8m - n}{n}$ equals ,

(A) $\frac{8m}{n}$

(B) $8m - 1$

(C) $8m - n$

(D) $\frac{8m}{n} - 1$

$\frac{27}{2}$

You did not make the right choice. Let us do a part of this problem together:

$$x + 3 \overline{)\ 3x^2 + 7x - 6}^{\ 3x}$$
$$\underline{3x^2 + 9x}$$

At this point, you should keep in mind that $(3x^2 + 9x)$ is subtracted from $(3x^2 + 7x)$. When we subtract two polynomials, we change the signs of the terms of the polynomial being subtracted, the subtrahend, and then apply the addition rules.

Please return to page $\frac{22}{2}$ and try this question again.

28/1

Each of the terms in the numerator are different since they have different powers of b. The first term has the letter b raised to the 4^{th} power; the second term has the letter b raised to the 3^{rd} power; and the last term has b raised to the 2^{nd} power. Therefore, these are unlike terms and cannot be combined. It would help you if you re-read the text assignment.

Return to page 11/2 and try this question again.

28/2

This choice is correct.

Now proceed to question 12 which follows:

Question 12

Perform the indicated calculation to find the value of the quotient

$$\frac{8x^3 - 4x^2 - 2x}{2x}$$, when $x = 2$, and select the letter next to the correct statement.

(A) 20

(B) 12

(C) 22

(D) 11

Let us explain how an expression of this type can be simplified.

Consider the expression

$$\frac{x^3y^2 + x^2y + y}{y}$$

This expression can be written as,

$$\left(\frac{1}{y}\right)\left(x^3y^2 + x^2y + y\right)$$

By the distributive property, the above equals,

$$\frac{x^3y^2}{y} + \frac{x^2y}{y} + \frac{y}{y}$$

Using the rule for finding the quotient of powers, we get

$$\frac{x^3y^2}{y} = x^3y \quad , \quad \frac{x^2y}{y} = x^2 \quad , \quad \frac{y}{y} = 1$$

Note that the last term is a number divided by itself and is, therefore, equal to 1.

Please return to page 15/1 and try this question again.

In a Long division problem where the dividend consists of many terms many operations have to be performed. We subtract and multiply quite a few times. Hence, there are more chances for making mistakes. In a situation like this, it is very important to work carefully and arrange your work neatly. Furthermore, when you have completed such a problem it is important to check your answer. You should multiply the quotient by the divisor and see if the product equals the dividend.

Please return to page 31/2 and try this question again.

30/1

Congratulations, you made the right choice. Here is how this question can be answered.

$$\frac{x + y}{y} = 3$$

$\frac{x}{y} + \frac{y}{y} = 3$ Now, $\frac{y}{y} = 1$ Hence, we have the equation

$$\frac{x}{y} + 1 = 3$$

$$\frac{x}{y} = 2 \text{ , or}$$

$$\frac{x}{y} = \frac{2}{1}$$

You have now completed this segment, hand in the PUNCH CARD. Make sure that you have written the following items in your notebook.

1. A problem worked out in detail involving the quotient of a polynomial and a monomial.

   For example: $\frac{8x^3y - 16xy^2 + 4x^2y}{4x^2y}$

You should now do questions 1-4 of the homework assignment.

30/2

Volume 8 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you you are asked to punch out the following:

COLUMNS	48 and 50	3	7	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	0	8	(Volume Number)
	66 and 68	0	2	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 221 - 223.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

In the following division, $645 \div 23 = 228\ \frac{1}{23}$, recognize which is the dividend, divisor, quotient, and remainder. Select the letter next to the correct statement.

	dividend	divisor	quotient	remainder
(A)	645	23	28	$\frac{1}{23}$
(B)	23	645	28	$\frac{1}{23}$
(C)	645	23	28	1
(D)	28	23	645	1

This choice is correct.

Now proceed to question 8 which follows:

Question 8

Apply the proper principle to find the quotient,

$$(2x^3 - 3x^2 + 5x - 2) \div (2x - 1) , \text{ and}$$

select the letter which labels the correct answer.

(A) $x^2 + 2$

(B) $x^2 - 2x + \frac{7}{2}$ remainder $-5\frac{1}{2}$

(C) $x^2 + x - 2$ and remainder 4

(D) $x^2 - x + 2$

You did very well until the last step. Please note that in the final step of this problem, you have to subtract the 2 binomials,

$$\begin{array}{r} 15x + 8 \\ \underline{15x - 20} \end{array}$$

Examine your work and see if you can find the error you made.

Return to page $\frac{36}{2}$ and try this question again.

$\frac{32}{1}$

a^2 is followed by "a" , and that is descending order. However, 4 is of a lower order than "a" . It could be written $4a^0$. At this point in the course, a^0 could be interpreted to mean " no a " ; therefore, the 4 is out of order.

Return to page $\frac{25}{2}$ and answer the question again.

$\frac{32}{2}$

It is true that $\frac{3(231x)}{3} = \frac{693x}{3}$. However, you were asked to chose a <u>polynomial</u> of three terms divided by monomial. 3(231x) is a monomial.

Please return to page $\frac{26}{2}$ and answer the question again.

$\frac{33}{1}$

You made a mistake. It is always a good idea to check your answer. If your choice were correct, then $(x - 9) (x - 2)$ would be equal to the dividend, $x^2 - 7x + 10$. Multiply these two binomials and see that their product does not equal the dividend.

Return to page $\frac{37}{1}$ and try this question again.

$\frac{33}{2}$

We do not agree. Please note that the dividend has two missing terms. It should be re-written as

$x^3 + 0x^2 + 0x - 8$ since an x^2 term and an "x" term will appear in the division process.

Do this question over again; and when you have completed the division, check your answer by multiplying the quotient by the divisor. This product should equal the dividend.

Return to page $\frac{47}{2}$ and choose another letter.

$\frac{34}{1}$

This choice is correct.

Now proceed to question 10 which follows:

Question 10

Apply the proper principle to find the quotient,

$$\frac{3x + 2x - 2}{2x - 1}$$

and select the letter which labels the correct answer.

(A) $\frac{3}{2} + x$ and remainder $x - \frac{1}{2}$

(B) $\frac{4x - 2}{-1}$

(C) $x + 2$

(D) $x + 1$ and remainder -3

$\frac{34}{2}$

You cancelled individual terms in the numerator with individual terms in the denominator. You can only cancel like factors in the numerator with like factors in the denominator. Thus, for example, in the quotient

$$\frac{x^4 + x^2 + 1}{x^2 + x + 1},$$

you cannot cancel the x^2 terms. Go back to the text assignment and study the procedure for dividing two polynomials.

Return to page $\frac{40}{2}$ and try this question again.

$\frac{35}{1}$

We do not agree. We are sure you noticed that there is a missing term in the dividend. Please re-write the dividend as

$4x^2 + 0x - 1$, since an x term will appear in the division.

Also, make sure that in subtracting, you change the sign of the lower term and add.

Please return to page $\frac{53}{2}$ and try this question again.

$\frac{35}{2}$

We do not agree. It seems that the letter "a" is the cause of your difficulty. Well, it shouldn't be. Proceed as you would in any problem in division, as follows,

$$x - 1 \overline{)\ x^2 + 2x - a}$$

Eventually, you will get a remainder expressed in terms of the letter "a". Since we are told that the remainder is zero, you can equate the expression for the remainder to zero. Solving the equation will lead us to the correct answer.

Please return to page $\frac{50}{2}$ and try this question again.

$\frac{36}{1}$

Very good. You made the correct choice. Here is one way the problem can be done.

$$
\begin{array}{r}
x^2 - x + 2 \\
2x - 1 \;\overline{)\; 2x^3 - 3x^2 + 5x - 2} \\
\underline{2x^3 - x^2} \\
-2x^2 + 5x \\
\underline{-2x^2 + x} \\
4x - 2 \\
\underline{4x - 2} \\
\end{array}
$$

Please proceed to question 9 below.

$\frac{36}{2}$

Question 9

Apply the proper principle to perform the indicated operation:

$$(6x^2 + 7x + 8) - (3x - 4)$$, and

select the letter next to the correct answer.

(A) $2x + 5 + \frac{28}{3x}$ and remainder $\frac{112}{3x}$

(B) $2x + 5$ and remainder 28

(C) $2x - 5$ and remainder -12

(D) $2x + 5$ and remainder $\frac{28}{3x-4}$

This choice is correct.

Now proceed to question 6 which follows:

Question 6

Apply the proper principle to find the quotient,

$(x^2 - 7x + 10) \div (x - 2)$ and select the letter

which labels the correct answer.

(A) $x + 5$

(B) $x - 9$

(C) $x - 9$ and remainder 8

(D) $x - 5$

$\frac{38}{1}$

Before you start to divide two polynomials, you must first make sure that both the dividend and the divisor are written in descending order of the variable. In this question, the divisor should be written as $(x + 3)$. Furthermore, you made a mistake in subtraction of polynomials.

Let us illustrate with another example:

$$y + 5 \overline{)\ 5y^2 - 8y + 10}\ \Big)\ \begin{array}{l} 5y \\ \\ \underline{5y^2 + 25y} \end{array}$$

When you subtract $5y^2 + 25y$ from $5y^2 - 8y$, the difference is $-33y$.

Please refresh your memory with regard to the rule for subtracting two polynomials.

"Change the signs of the subtrahend and apply the rules for addition."

Return to page $\frac{22}{2}$ and try this question again.

$\frac{38}{2}$

You did not make the right choice. If you examine the dividend, you will notice that two facts must be attended to.

First, the dividend is not written in descending order of the letter x.

Second, the dividend has missing terms. The x^3 and the x terms are missing. Hence, before starting to divide, the dividend should be re-written as

$$x^4 + 0x^3 + x^2 + 0x + 1$$

Please continue from here.

Return to page $\frac{40}{2}$ and try this question again.

The choice you made indicates that you forgot to do something important. Before starting a problem involving the division of two polynomials, you should make sure that both the dividend and the divisor are written in descending order of the variable. Please examine the dividend and arrange it in descending order of " x "

Return to page $\frac{34}{1}$ and try this question again.

You did not make the right choice. Suppose we do a similar problem:

$$(9y^2 - 1) \div (3y - 1)$$

First, we notice that there is a missing term in the dividend. There is no " y " term. Therefore, we write the dividend as follows:

$$9y^2 + 0y - 1$$ We obtain

$$3y - 1 \overline{)\,9y^2 + 0y - 1}^{\,3y}$$

$$\underline{9y^2 - 3y}$$

Now, you must be careful. In subtracting $-3y$ from $0y$, the sign of the lower term changes, and we add $3y + 0y$.
We are sure that you can continue from here without our help.

Return to page $\frac{53}{2}$ and try this question again.

$\frac{40}{1}$

Very good. You made the correct choice. In this question, we first have to re-write the dividend to include the two missing terms. Thus, we write

$$x^3 - 8 = x^3 + 0x^2 + 0x - 8$$

$$\begin{array}{r} x^2 + 2x + 4 \\ x - 2 \;\overline{)}\; x^3 + 0x^2 + 0x - 8 \\ \underline{x^3 - 2x^2} \\ 2x^2 + 0x - 8 \\ \underline{2x^2 - 4x} \\ 4x - 8 \\ \underline{4x - 8} \\ 0 \quad \text{remainder} \end{array}$$

Please proceed to question 13 below.

$\frac{40}{2}$

Question 13

Apply the proper principle to find the quotient,

$(x^2 + x^4 + 1) - (x^2 + x + 1)$, and select the letter which labels the correct statement.

(A) x^3

(B) $x^2 - x + 1$

(C) $x^2 + x - 1$ and remainder 1

(D) None of these.

Volume 8 Segment 3 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you you are asked to punch out the following:

COLUMNS	48	and	50	3	8 (Sequence Number)
	54	and	56	0	4 (Type of Punch Card)
	60	and	62	0	8 (Volume Number)
	66	and	68	0	3 (Segment Number)

SUPPLEMENTARY NOTES

What is factoring?

In the book, Alice in Wonderland, the Mad Hatter makes the point that it would be more fun to celebrate one's 364 unbirthdays rather than the single birthday. Don't you agree? In your study of mathematics, you have been asked to multiply and multiply. Now, for the first time, you will be asked to "unmultiply". Let us explain. If you are given the numbers, say, 3,5,7, and you are asked to find their product, you get $3 \times 5 \times 7 = 105$; this is multiplying. If on the other hand you are given the number 105, and you are asked to find the numbers 3,5,7 whose product will equal 105; this is unmultiplying. However, instead of the word unmultiplying, we use the word factoring. The numbers 3,5,7 are called the factors of 105.

Why do we factor?

The process of factoring comes under the heading of "tricks of the trade." Just as craftsmen, such as carpenters, electricians, or plumbers, have ways of doing things that make their job easier, so the mathematician uses factoring to solve certain problems with much less effort. Hence, if you want to become a good craftsman in algebra, you must learn how to factor.

Your READING ASSIGNMENT for this Segment is pg: 237 - 244
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Perform the necessary operation to express each of the numbers 210, 252 as a product of numbers and select the letter which labels the correct statement.

(A) $210 = 7 \times 6 \times 2 \times 5$
$252 = 4 \times 7 \times 6 \times 2$

(B) $210 = 7 \times 5 \times 3 \times 2$
$252 = 21 \times 2 \times 3 \times 3$

(C) $210 = 7 \times 5 \times 3 \times 2$
$252 = 7 \times 3 \times 2 \times 3 \times 2$

(D) None of these.

$\frac{42}{1}$

You went too far. A good question to ask is: when is the operation of division completed? In other words, when do we stop dividing? The answer to this question is very important.

The operation of division is completed when the remainder is zero; or, when the remainder is of a degree less than that of the divisor.

Suppose we do a problem to illustrate the above statement.

$$
\begin{array}{r}
2x - 1 \\
4x - 1 \;\overline{)\; 8x^2 - 6x + 3} \\
\underline{8x^2 - 2x} \\
-4x + 3 \\
\underline{-4x + 1} \\
2 \quad \text{remainder}
\end{array}
$$

The first remainder is $-4x + 3$. This remainder is of the same degree as the divisor. Hence, we have to continue. The second remainder is 2, which is of lower degree than the divisor. Therefore, we stop dividing.

Please return to page $\frac{36}{2}$ and try this question again.

$\frac{42}{2}$

You are not playing the game according to the rules. You are not allowed to cancel individual terms in the numerator of a fraction with individual terms in the denominator. Only equal factors can be cancelled. Thus, $\frac{2x^2yz}{2y}$ is a product of numbers divided by a product of numbers and the 2's and y's can be cancelled. However, in the quotient, $\frac{2x^2 + y - z}{2x^2 + y}$, you <u>cannot</u> cancel any of the like terms in the numerator with the like terms in the denominator. Go back to your text assignment, and study the procedure for dividing two polynomials.

Return to page $\frac{34}{1}$ and try this question again.

You are correct.

A polynomial written in decreasing order of a variable means that the exponents of the variable get smaller as you go from left to right. All we are concerned with is the exponents, not the size of the coefficients. For example, the following polynomial is written in decreasing order of the variable r ;

$$3r^4 + r^3 - 50r^2 + 75r + 100$$

You should also note that a number without a letter is of lower order than a number with a letter. Thus, 100 is of lower order than 75r since 100 could be written as $100r^0$

At this point in the course, r^0 could be interpreted as "no r ".

Please proceed to question 4 below.

Question 4

Recognize the missing terms that must be inserted in the polynomial

$$5x^4 - x^2 + 1$$

to make it a complete polynomial of order 4 .

Select the letter which labels the correct answer.

(A) x^3 , x

(B) x^5 , x^3

(C) x^3

(D) x

$\frac{44}{1}$

It looks as if you forgot how to divide.

Perhaps you should review the textbook instruction on Page 222. In the long division method you have to subtract. Remember, when you subtract, you must change the signs of the subtrahend and then use the addition rules.

Return to page $\frac{37}{1}$ and try this question again.

$\frac{44}{2}$

You cancelled individual terms in the numerator with individual terms in the denominator. We cannot do this. You may ask, " why not? " Let us illustrate " why not " with an example.

Consider the quotient

$$\frac{4x^2 - 6}{2x - 3}$$

and let us find the value of this quotient when $x = 2$.

We get,

$$\frac{4(2^2) - 6}{2(2) - 3} = 10$$

Had we cancelled as you did, the quotient would have become $2x + 2$, which equals $\underline{6}$ when $x = 2$.

Please return to page $\frac{52}{2}$ and try this question again.

You probably made a mistake in your work, and your answer does not match any of the given choices. However, one of the given choices is a correct answer to this question. Let me help you get started. The dividend has two things wrong with it. It is not written in descending order of x, and there are missing terms. Re-write the dividend as

$$x^4 + 0x^3 + x^2 + 0x + 1$$

and do this problem over again.

Return to page 40/2 and work out the problem again.

Although 48 does equal

$$4 \times 4 \times 3$$

these are not the <u>prime</u> factors of 48 ; the number 4 is not a prime number since it can be expressed as the product of two smaller numbers.

$$4 = 2 \times 2 .$$

4 is a composite number.

Return to page 78/1 and try this question again.

$\frac{46}{1}$

Good! You did a strange problem correctly.

Check through this method:

$$
\begin{array}{r}
x + 3 \\
x - 1 \overline{)\ x^2 + 2x - a} \\
\underline{x^2 - x} \\
3x - a \\
\underline{3x - 3} \\
3 - a
\end{array}
$$

Now, we were told that the remainder is zero!

Therefore, $3 - a$ must equal zero.

That is, $3 - a = 0$

or, $3 = a$

Check this by multiplying $(x - 1)$ by $(x + 3)$

and see if the result is $x^2 + 2x - (3)$.

$\frac{46}{2}$

You have now completed this segment. Hand in your PUNCH CARD. Make sure that the following items are in your notebook.

(1) Definition of the words dividend, divisor, quotient, and remainder.

(2) The identity $\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$

(3) A question involving the quotient of two polynomials worked out in detail; for example,

$$(4x^2 + x - 5) \div (2x - 1)$$

You should now do question 5 - 8 of the homework assignment.

You added when you should have subtracted. Thus,

$$2x - 1 \;\overline{)\; 2x^2 + 3x - 2}\;\Big|\; x$$
$$\underline{2x^2 - x}$$

$(2x^2 - x)$ is subtracted from $2a^2 + 3x)$. The rule for subtracting 2 polynomials is to change the signs of the terms of the lower polynomial and add.

Return to page 34 and try this question again.

This choice is correct.

Now proceed to question 12 which follows.

Question 12

Apply the proper principle to find the quotient, $(x^3 - 8) - (x - 2)$, and select the letter which labels the correct answer.

(A) $x^2 - 4$

(B) $x^2 + 2x + 4$

(C) $x^2 + 4x + 4$

(D) x^2 and remainder $\frac{2x^2 - 8}{x - 2}$.

You chose the correct answer. The procedure for dividing two polynomials is carefully explained in the text assignment, Page 222. However, since the division of polynomials is a process consisting of many steps, let's go over this procedure very carefully, one step at a time and do this problem together.

$$(2x^2 + 5x - 3) - (x + 3)$$

Steps

(1) We re-write the quotient as a problem in long division.

$$x + 3 \overline{)\ 2x^2 + 5x - 3}$$

(2) We find the quotient of the first term of the dividend and the first term of the divisor, $\frac{2x}{x} = 2x$. This becomes the first term of the quotient and is written over the division sign.

$$x + 3 \overline{)\ \begin{array}{l} 2x \\ \hline 2x^2 + 5x - 3 \end{array}}$$

(3) Next, we multiply $2x(x + 3) = 2x + 6x$, and write this under the first two terms of the dividend. Are you still with us?

$$x + 3 \overline{)\ \begin{array}{l} 2x \\ \hline 2x^2 + 5x - 3 \\ 2x^2 + 6x \end{array}}$$

(4) We now subtract, getting $-x\ -3$.

$$x + 3 \overline{)\ \begin{array}{l} 2x \\ \hline 2x^2 + 5x - 3 \\ 2x^2 + 6x \\ \hline -\ x - 3 \end{array}}$$

(5) As before, we consider the quotient $\frac{-x}{x}$ which equals -1 and write this number above the division sign.

$$x + 3 \overline{)\ \begin{array}{l} 2x - 1 \\ \hline 2x^2 + 5x - 3 \\ 2x^2 + 6x \\ \hline -\ x - 3 \end{array}}$$

(6) We multiply $(-1) (x + 3)$ and subtract again. As you can see, there is no remainder; hence,

$$x + 3 \overline{)\ \begin{array}{l} 2x - 1 \\ \hline 2x^2 + 5x - 3 \\ 2x^2 + 6x \\ \hline -\ x - 3 \\ -\ x - 3 \\ \hline 0 \text{ remainder} \end{array}}$$

$2x - 1$ is the answer.

(7) Now, to make sure, check the division by multiplying $(2x - 1) (x + 3)$. This product should equal the dividend $2x^2 + 5x - 3$.

$$\begin{array}{r} x + 3 \\ \hline 2x - 1 \\ \hline 2x^2 + 6x \\ -\ x - 3 \\ \hline 2x^2 + 5x - 3 \end{array}$$

Copy this page into your notebook.
Proceed to question 7 on page $\frac{22}{2}$

We do not agree. If we subtract (-2x - 6) from (-2x - 6), the difference is zero. The rule for subtracting two polynomials is to change the signs of the terms of the subtrahend and apply the rules for addition.

There is one other item that needs clearing up. You confused the remainder with the quotient of the remainder and the divisor. It will help you to remember this formula:

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} = \frac{\text{REMAINDER}}{\text{DIVISOR}} .$$

Note that the word remainder appears only in the numerator.

Please return to page $\frac{22}{2}$ and try this question again.

The quotient is right; the remainder is wrong. Recall the formula.

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} = \frac{\text{REMAINDER}}{\text{DIVISOR}}$$

Note that the remainder is <u>not</u> a quotient.

Please return to page $\frac{36}{2}$ and try this question again.

$\frac{50}{1}$

Very good. Check this method against your work.

$$
\begin{array}{r}
x^2 - x + 1 \\
\hline
x^2 + x + 1 \;\big)\; x^4 + 0x^3 + x^2 + 0x + 1 \\
\underline{x^4 + x^3 + x^2} \\
-x^3 + 0x^2 + 0x + 1 \\
\underline{-x^3 - x^2 - x} \\
x^2 + x + 1 \\
\underline{x^2 + x + 1} \\
0
\end{array}
$$

Proceed to question 14 below.

$\frac{50}{2}$

Question 14

You are told that when $x^2 + 2x - a$ is divided by $x - 1$, there is no remainder. Do the division and determine a value of the letter a. Select the correct answer.

(A) $a = 0$

(B) $a = 3$

(C) $a = 1$

(D) Cannot be determined.

$\frac{51}{1}$

We don't agree. Let me help you get started. First consider the number 238 and express it as the product of prime numbers. Do the same to the number 255. Examine the factors of each of these numbers and select the largest factor that they have in common.

Please return to page $\frac{59}{2}$ and try this question again.

$\frac{51}{2}$

You made one error. If you examine the middle term of the given polynomial, you will notice that the letter "c" is raised to the first power. The greatest common factor cannot have the letter "c" raised to a larger power.

Please return to page $\frac{58}{2}$ and try this question again.

$\frac{52}{1}$

We do not agree. In order to find the factors of a number, say 252, we first observe that this number is even. Hence, 2 is a factor and $\frac{252}{2} = 126$. Now, 126 is still even, and we can divide by 2 again $\frac{126}{2} = 63$.

Clearly, 63 can be divided by 3 and the quotient is 21.

We are sure that you can continue from here. You should note that with each division the number gets smaller, and it is easier to discover factors of smaller numbers.

Please return to page $\frac{41}{2}$ and try this question again.

$\frac{52}{2}$

We do not agree. If you raise $2x$ to the 3^{rd} power, both 2 and x are each raised to this power.

Thus, $(2x)^3 = 2^3 . .x3 = 8x^3$.

Clearly, 8 does not divide 12 exactly.

Please return to page $\frac{61}{2}$ and try this question again.

Very good. You made the correct choice.

Let us do this problem together, step by step.

$2x - 1 \overline{)\ 3x + 2x^2 - 2}$ — rearrange in descending order

$2x - 1 \overline{)\ 2x^2 + 3x - 2}$ — $\frac{2x^2}{2x} = x$

$2x - 1 \overline{)\ \begin{array}{r} x \\ 2x^2 + 3x - 2 \end{array}}$ — multiply divisor $(2x - 1)$ by quotient (x)

$$\begin{array}{r} x \\ 2x - 1 \overline{)\ 2x^2 + 3x - 2} \\ \underline{2x^2 - x} \end{array}$$ — subtract

$$\begin{array}{r} x \\ 2x - 1 \overline{)\ 2x^2 + 3x - 2} \\ \underline{2x^2 - x} \\ 4x - 2 \end{array}$$ — $\frac{4x}{2x} = 2$

$$\begin{array}{r} x + 2 \\ 2x - 1 \overline{)\ 2x^2 + 3x - 2} \\ \underline{2x^2 - x} \\ + 4x - 2 \end{array}$$ — multiply divisor $(2x - 1)$ by 2

$$\begin{array}{r} x + 2 \\ 2x - 1 \overline{)\ 2x^2 + 3x - 2} \\ \underline{2x^2 - x} \\ + 4x - 2 \\ \underline{4x - 2} \\ 0 \end{array}$$ — subtract; remainder = 0

Proof: $(2x - 1)(x + 2) = 2x^2 + 3x - 2$

Please go on to question 11 below.

Question 11

Apply the proper principle to find the quotient,

$(4x^2 - 1) - (2x - 1)$, and select the letter which labels the correct answer.

(A) $2x - 1$ and remainder $- 2$

(B) $2x - 1$

(C) $2x$

(D) $2x + 1$

$\frac{54}{1}$

You did not make the correct choice. In order to find the factors of a number, we start with dividing this number by the smallest divisor first. Thus, we first try dividing by 2:

$$\frac{246}{2} = 123.$$

Then we try dividing by 3 and so on.

We are sure that you can continue from here by yourself.

Return to page $\frac{66}{2}$ and try this question again.

$\frac{54}{2}$

This choice is correct.

Now proceed to question 8 which follows.

Question 8

Apply the proper principle to find the greatest common monomial factor of the monomials: s: $7r^2 s^2 t^2$ and $-21r^3 st^2$. Then select the letter which labels the correct answer.

(A) $-3rst$

(B) $7r^2st^2$

(C) $-7r^2st^2$

(D) $3r^2st^2$

$\frac{55}{1}$

You made the wrong choice.

You have to consider only the numbers 60 and 48 . Express each of these numbers as a product of primes. After having done this, examine the factors to see what numbers they have in common.

Please return to page $\frac{62}{2}$ and try this question again.

$\frac{55}{2}$

You did not make the right choice. In order for c^3 to be a factor each of the terms of the given polynomial must have the letter "c" raised to the third power or better.

Since this is not the case, please reconsider your choice.

Return to page $\frac{58}{2}$ and try this question again.

56/1

Although 84 does equal 21 x 2 x 2 , this is not the correct answer; the question called for prime factors.

The number 21 is not prime, since it can be written as a product of 2 other numbers.

Thus, $21 = 7 \times 3$.

A prime number can be written only as a product of 1 and itself.

For example, $19 = 1(19)$.

Please return to page 78/1 and try this question again.

56/2

If you raise 3x to the third power, both 3 and x are each raised to this power.

Thus, $(3x)^3 = 3^3 \cdot x^3 = 27x^3$.

We are sure that you can see that the number 27 is too large to be a factor of 12 .

Please return to page 61/2 and try this question again.

We do not agree.

It is true that 5 is a factor of 45 and 15 . However, in trying to find the greatest common monomial factor of two monomials, first determine the largest common factor of the numerical coefficients. Examine the numbers 45 and 15 more carefully and decide what their largest common factor is.

Please return to page 60/2 and try the question again.

We do not agree. To check whether you factored correctly, the product of the two factors should equal the original polynomial. Thus,

$$7 (2x - 3y) = 14x - 21y ,$$

and this is not the original polynomial. You are missing the number 7 . Now, ask yourself, "what number times 7 equals 7 ." Certainly 7 x 0 does not equal 7 .

Return to page 69/2 and try this question again.

58
1

Very good. You made the correct choice. Here is how this question can be answered. We first consider the numerical coefficients, and we find that 7 is the largest common factor of 7 and 21 . Then we see that r^2 is the largest power of r that is common to both monomials. Similarly, the largest power of s is s , and the largest power of t is t^2 . Hence the answer is $7r^2 st^2$.

Please go on to question 9 below.

58
2

Question 9

Apply the proper principle to find the greatest common monomial factor of the polynomial, $5c^3 d - 10cd^2 + 15c^2 d^2$.

Then, select the letter which labels the correct answer.

(A) 5cd

(B) $5c^2d$

(C) $5c^3d^2$

(D) None of these.

Very good. You made the correct choice. Using the algorithm for finding the prime factors of a number,

$$\begin{array}{r|l} 2 & 246 \\ \hline 3 & 123 \\ \hline & 41 \end{array}$$

We first divide the given number by 2 $\frac{246}{2} = 123$

Now we divide the quotient by 3 $\frac{123}{3} = 41$

Clearly, 41 is a prime number

Please go on to question 5 below.

Question 5

Perform the necessary computation and select the letter which correctly completes the statement: The largest common factor of 238 and 255 is:

(A) 3

(B) 7

(C) greater than 13, but less than 24.

(D) greater than 24, but less than 51.

60
1

Very good. You made the correct choice. Here is how this question can be answered. We express each of the numbers 60 , 48 as a product of primes; thus,

$$60 = \overline{2 \times 2 \times 3} \times 5$$

$$48 = 2 \times 2 \times 2 \overline{\times 2 \times 3}$$

Now we match like factors of the first number with the second number as indicated by the lines over the numbers. We see that $2 \times 2 \times 3 = 12$ match. Hence, the largest numerical coefficient that is a factor of both binomials is 12 .

Please go on to question 7 below.

60
2

Question 7

Apply the proper principle to find the greatest common monomial factor of the monomials, $45x^3y^2$ and $15x^2y^5$. Then, select the letter which labels the correct answer.

(A) $5x^2y^2$

(B) $9x^2y^2$

(C) $15x^3y^5$

(D) $15x^2y^2$

Although 150 does equal 25 x 3 x 2 , this is not the correct answer. The question called for prime factors.

The number 25 is not a prime number since it can be expressed as the product of two smaller numbers. Thus, 25 = 5 x 5 . Any number that can be expressed as a product of other numbers is called composite. For example, 66 is a composite number, since

$$66 = 2 \times 3 \times 11$$

Please return to page 78/1 and try this question again.

This choice is correct.

Now proceed to question 10 which follows.

Question 10

Apply the proper principle to find the highest power of the first monomial that is a factor of the second monomial.

$$2x, \quad 12x^3y^2 ,$$

and select the letter which labels the correct answer.

(A) $(2x)^3$

(B) $(2x)^2$

(C) $((3x)^3$

(D) None of these.

62
1

Very good. You made the correct choice. Let us do this question together. First, we take 238 and write it as a product of primes: thus,

$$238 = 2 \times 7 \times 17$$

Now, we work with 255. We start by dividing this number by 3 . We find

$$255 = 3 \times 5 \times 17$$

and the largest common factor of two given numbers is, therefore, 17 . 17 is greater than 13 and less than 24 .

Please go on to question 6 below.

62
2

Question 6

Perform the necessary computation and select the letter which correctly completes the statement: The largest number that is a factor of the two monomials

$60y^3z$ and $48y^2z^2$ is:

(A) 6

(B) 12

(C) 16

(D) 30

Very good. You made the right choice. Here is how we proceed to express a number as a product of other numbers.

We start with the smallest possible division of the number.

A convenient form (or algorithm) is illustrated here:

a) $2 \mid \overline{210}$ Thus, $\frac{210}{2} = 105$
$\quad\;\; \overline{105}$

b) $2 \mid \overline{210}$ Now, a number ending in 5 is
$5 \mid \overline{105}$ divisible by 5, $\frac{105}{5} = 21$
$\quad\;\; \overline{21}$

c) $2 \mid \overline{210}$ and $\frac{21}{3} = 7$
$5 \mid \overline{105}$
$3 \mid \overline{21}$
$\quad\;\; \overline{7}$

d) Hence, $210 = 7 \times 5 \times 3 \times 2$

and the factors of 252 can be found a similar way.

$2 \mid \overline{252}$
$2 \mid \overline{126}$
$3 \mid \overline{63}$
$3 \mid \overline{21}$
$\quad\;\; \overline{7}$

$252 = 7 \times 3 \times 3 \times 2 \times 2$

Please go on to question 2 below.

Question 2

Recognize which of the following represent a factoring of the number 90 over the set of positive integers and select the letter which labels the correct choice.

(A) $9 \times 5 \times 4 \times \frac{1}{2}$

(B) $9 \times 5 \times 2$

(C) $(-1) (-2) (5) (9)$

(D) $(18) (5) (\frac{1}{9}) (9)$

$\frac{64}{1}$

We do not agree. One of the choices contains the correct answer.

Please return to page $\frac{58}{2}$ and try this question again.

$\frac{64}{2}$

You did not make the correct choice. One of the letters has the correct answer next to it. It may be helpful if you recall the rule:

$$(ab)^n = a^n b^n .$$

For example,

$$(5y)^2 = 5^2 \cdot y^2 = 25y^2$$

Please return to page $\frac{61}{2}$ and try this question again.

A good way to proceed in answering this question is to start from the left and work your way to the right. First find the largest common factor of 7 and 21 . Then consider the letter "r" and find the largest power of this letter that is common to both monomials. Proceed and do the same for the letters "s" and "t" .

Please return to page 54/2 and try this question again.

If you multiply the two factors that you chose;

namely, $3ab$ and $ab - 3b$,

you will find that this product does not equal the original polynomial.
Hence, you did not make the right choice.
The following illustration should be helpful.

Write $x^3 + x^2 + x$

in factored form.

Since x divides each of the above terms, x is a factor; we can write,

$$x (- - -).$$

To fill in the dashes you should ask yourself,

"what must multiply x to give me x^3 ?"

The answer is $x \; x^2 = x^3$.

Next you ask yourself,

"what must multiply x to give me x^2 ?"

The answer is $x \; x = x^2$.

Finally,

what must multiply x to give x ?

The answer is $x \; 1 = x$.

Hence, the answer is

$$x (x^2 + x + 1)$$

Please return to page 79/2 and try this question again.

66
1

Your choice,

$231 = 3 \times 11 \times 7$ is the correct answer;

the question specified prime factors.

A prime number is a number whose only whole number divisors are 1 and the number itself. There is one exception; the number 1 is not considered a prime.

For future reference, we will write
the first 10 prime numbers,

2, 3, 5, 7, 11, 13, 17, 19, 23, 29 .

Note that each of these numbers has no exact divisors except 1 and the number itself. You should write these in your notebook.

Go on to question 4 below.

66
2

Question 4

Perform the necessary computation and select the letter which completes the following statement correctly.

The largest prime factor of 246 is:

(A) 3

(B) 13

(C) greater than 13 but less than 32.

(D) greater than 32 but less than 42.

To check a factorization you just have to multiply the two factors and see whether their product is the original polynomial.

Thus $7 (2x - 3y + 1) = 14x - 21y + 7 .$

Therefore, you are correct.

Proceed to question 12 below.

Question 12

Apply the proper principle and write in factored form,

$$5p^2 q - 10pq^2 .$$

Select the letter which labels the correct answer.

(A) $5pq (p - 2q)$

(B) $5pq (p - 2)$

(C) $5p (pq - 2q^2)$

(D) $5q (p^2 - 2pq)$

$\frac{68}{1}$

The number 9 is an exact divisor of 45 , but it does not divide 15 exactly. You are looking for the largest number that will divide both 45 and 15 exactly.

Please return to page $\frac{60}{2}$ and try this question again.

$\frac{68}{2}$

You misunderstood. Writing an expression in factored form means writing it as a product of simpler expressions. You cleared parenthesis and combined like terms; that is not factoring. Examine the question to see if there is an expression that is common to both terms. This expression is one of the factors. Can you find the other?

Please return to page $\frac{81}{2}$ and try this question again.

Since

$$(2x)^2 = 4x^2$$

we must first check to see if $4x^2$ is a factor of $12x^3y^2$; that is, does $4x^2$ divide into $12x^3y^2$ evenly.

i.e. $4x^2 \overline{)\ 12x^3y^2}$ $\quad 3xy^2$ so it is a factor.

Then we check to see if $2x$ is the highest power that divides $12x^3y^2$ evenly. We try $(2x)^3$ which equals $8x^3$. But $8x^3$ doesn't divide into $12x^3y^2$.

Therefore, your choice, $(2x)^2$ is correct.

Proceed to question 11 below.

Question 11

Apply the proper principle and write the polynomial

$$14x - 21y + 7$$

as the product of a numerical coefficient and a polynomial.

Select the letter which labels the correct statement.

(A) $7(2x - 3y)$

(B) $7x(2 - 3y)$

(C) $7(2x - 3y + 1)$

(D) None of these.

$\frac{70}{1}$

The product $3ab\ (\ a\ -\ 5b\ )\ =\ 3a^2b\ -\ 15ab^2$. This product should equal the original polynomial. Since this is not the case, please reconsider your choice.

Return to page $\frac{79}{2}$ and try this question again.

$\frac{70}{2}$

You must arrange your work in an organized fashion. Start with the situation: The shaded area is the area of the rectangle minus the area of the semi-circle. Now, here are some of the questions you must answer:

What is the formula for the area of a rectangle? $A = l \cdot w$

What is the formula for the area of a circle? $A = \pi r^2$

What is the formula for the area of a semi-circle?

Return to page $\frac{77}{2}$ and try this question again.

If you examine the first two terms of the question; namely,

$y^2 - y$ you will notice that they can

be written in factored form as

$y(y - 1)$

If you examine the remaining two terms,

$yz - z$ you will notice that they can

also be written in factored form as

$z(y - 1)$

Please continue from here.

Return to page $\frac{86}{2}$ and try this question again.

You did not answer the question. The slide rule is a very fine instrument, you were asked to simplify the computation by using factoring.

Please return to page $\frac{84}{2}$ and try this question again.

$\frac{72}{1}$

To remove the highest common monomial factor, it is necessary to examine the numerical coefficients first, and find the largest number which will divide both these coefficients exactly. This part of the question you did correctly. Then, the letters should be examined and the highest power of each letter that is common to all terms should be removed. For example, in the polynomial

$$x^3 y^5 + x^4 y^4 + x^2 y^7$$

the highest power of x that can be removed is x^2, because that is the highest power of x that is common to all the terms of the polynomial. Similarly, the highest power of y that can be removed is y^4.

Please return to page $\frac{67}{2}$ and try this question again.

$\frac{72}{2}$

The square of a number means the number multiplied by itself. For example, the square of 6 is

$$6 \times 6 = 36$$

The square of 9 is

$$9 \times 9 = 81$$

Please do not confuse the square of a number with multiplying a number by 2.

Return to page $\frac{83}{1}$ and choose another letter.

If your choice were correct, then

$7x (2 - 3y)$ would equal the original polynomial.

Since this is not the case, please reconsider your choice.

Return to page $\frac{69}{2}$ and try this question again.

We do not agree. Your error may be due to careless reading of the question. Note that we have

$$x (x - 2) + 3 (x - 2)$$

This is in the form

$$xa + 3a$$

which equals

$$a (x + 3)$$

Please return to page $\frac{81}{2}$ and try this question again.

74/1

You did not make the right choice. Factoring a number over the set of positive integers means that only positive integers can be used as factors. This eliminates factors that are fractions and negative numbers. However, the factors do not have to be prime numbers.

Please return to page 63/2 and try this question again.

74/2

You did not make the correct choice. You are looking for the highest powers of both x and y, which are common to both the given monomial. Consider, for example, the two monomials, $a^5 b^3$ and $a^2 b^4$. The largest power of a that is common to both monomials is a^2. A larger power of a, say a^3, is a factor of the first monomial; but not of the second. Similarly, the highest power of b that is a factor of both monomials, is b^3.

Return to page 60/2 and try this question again.

$\frac{75}{1}$

Very good. You made the right choice. Suppose we do this question together.

The area of the shaded part is obtained by subtracting the area of the semi-circle from the area of the rectangle.

Area of shaded part =

= Area of Rectangle	- Area of Semi-Circle	Situation
= length x width	- $\frac{1}{2} \cdot \pi \cdot$ (radius)^2	Formula
= 2r . r	- $\frac{1}{2}\ \pi\ r^2$	Substitution
= $2r^2$	- $\frac{1}{2}\ \pi\ r^2$	Simplify
= r^2 (2 - $\frac{1}{2}\pi$)		Factor

Please go on to question 17 below.

$\frac{75}{2}$

Question 17

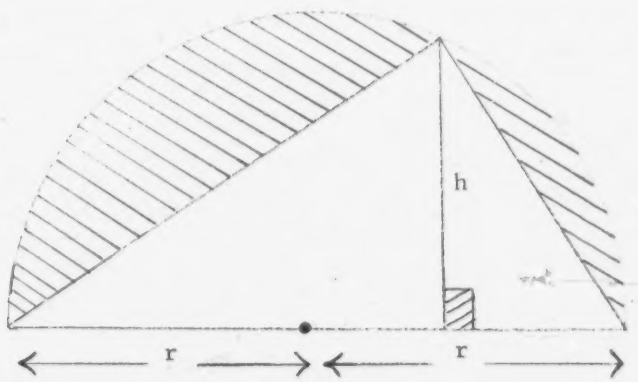
In the figure below, a triangle is inscribed in a semi-circle of radius r. Apply the proper principle to express the area of the shaded part in factored form. Select the letter which labels the correct answer.

(A) r ($\frac{\pi r}{2}$ - h)

(B) r (π r - h)

(C) r ($\frac{\pi}{2}$ - 2h)

(D) r (π - h)



$\frac{76}{1}$

Writing the polynomial

$3a^2b - 15ab^2 + 3ab$ in factored form means that

the largest common factor should be removed. Now, 3a does not fill this requirement. If you look at the trinomial of your answer, you will see that there is another letter that is common to all the three terms.

Please return to page $\frac{79}{2}$ and try this question again.

$\frac{76}{2}$

We do not agree.

Instead of multiplying large numbers like 98 , 54 , and then dividing, it is much easier to perform mathematical operations using smaller numbers. Furthermore, if you write the numerator and denominator as products of smaller numbers, many numbers may cancel, thus making your job much easier.

Please return to page $\frac{84}{2}$ and try this question again.

Very good. You made the right choice. This is a two-step problem. Here is how this question can be answered. We write the first two terms as

$$y (y - 1)$$

and the second two terms as

$$z (y - 1)$$

Therefore, $y^2 - y + yz - z = y (y - 1) + z (y - 1)$.

Now,

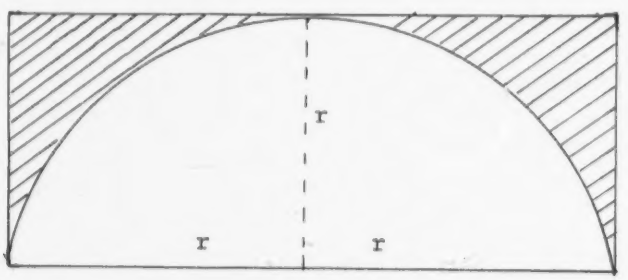
$(y - 1)$ is a common factor and can be removed. Hence, we get

$$(y - 1) (y + z).$$

Please go on to question 16 below.

Question 16

In the figure below, a rectangle is circumscribed about a semi-circle of radius r. Apply the proper principle to express the area of the shaded part in factored form and select the letter next to the correct answer.



(A) $r^2 (\frac{\pi}{2} - 2)$

(B) $r^2 (\pi - 2)$

(C) $r^2 (2 - \frac{\pi}{2})$

(D) $r^2 (3 - \pi)$

$\frac{78}{1}$

This choice is correct.

Now proceed to question 3 which follows:

Question 3

Recognize which of the following represent the number on the left side expressed as a product of prime factors, and select the letter next to the correct answer.

(A) $48 = 4 \times 4 \times 3$

(B) $84 = 21 \times 2 \times 2$

(C) $150 = 25 \times 3 \times 2$

(D) $231 = 3 \times 11 \times 7$

$\frac{78}{2}$

The correct answer is written next to one of the letters. Examine each of the coefficients 14 , 21 , 7 and choose the largest number that divides each of these 3 numbers exactly.

Please continue from here.

Return to page $\frac{69}{2}$ and try this question again.

We can always check a factorization by multiplying the factors.

$$5pq\ (\ p\ -\ 2q\ )\ =\ 5p^2q\ -\ 10pq^2$$

It is always advisable to check the factor chosen to see whether any other common factor can be removed.

Your answer is correct.

Proceed to question 13 below.

Question 13

Apply the proper principle and write the following polynomial in factored form:

$$3a^2b\ -\ 15ab^2\ +\ 3ab$$

Select the letter which labels the correct answer.

(A) $3ab\ (\ ab\ -\ 3b\ )$

(B) $3ab\ (\ a\ -\ 5b\ )$

(C) $3a\ (\ ab\ -\ 5b^2\ +\ b\ )$

(D) $3ab\ (\ a\ -\ 5b\ +\ 1\ )$

$\frac{80}{1}$

The area of the two shaded segments can be found by subtracting the area of the triangle from the area of the semi-circle.

The formula for the area of a triangle is $\frac{1}{2}$ b h

The formula for the area of a circle is π r^2

Please return to page $\frac{75}{2}$ and try this question again.

$\frac{80}{2}$

Not bad, you have the right idea.

However, you did not go far enough. By far enough, we mean that you should express each number as a product of prime numbers.

Please return to page $\frac{84}{2}$ and try this question again.

This choice is correct.

Now proceed to question 14 which follows:

Question 14

Apply the proper principle and write in this expression in factored form:

$$x (x - 2) + 3 (x - 2)$$

Select the letter which labels the correct answer.

(A) $x^2 + x - 6$

(B) $(x + 3) (2x - 4)$

(C) $4x (x - 2)$

(D) $(x - 2) (x + 3)$

We do not agree. Notice that the two binomials whose product we seek are exactly alike with the exception that the first binomial has a plus sign between its terms, while the second binomial has a minus sign. The product of such binomials can be written down immediately. It is the square of the first term minus the square of the second term.

Let us illustrate:

$$(3a + b) (3a - b) = 9a^2 - b^2$$

Please return to page 108/1 and try this question again.

$\frac{82}{1}$

You are right as far as you went.

$y^2 - y + yz - z$ does equal $y(y - 1) + z(y - 1)$

But this result can be factored further. Since the

$(y - 1)$ is common to both terms;

it, too, can be factored,

$(y - 1)(y + z)$

Return to page $\frac{86}{2}$ and try this question again.

$\frac{82}{2}$

We do not agree. First, let us make sure that we have the same definition of the square root of a number. If a number N is written as the product of two equal numbers, than each of the equal numbers is called the square root of the number N.

Thus; for example,

(1) if $81 = 9 \times 9$, then 9 is the square root of 81

(2) if $49a^2 = (7a)(7a)$, then $7a$ is the square root of $49a^2$

You can always check whether you have the correct square root by multiplying the square root by itself.

For example, if $\frac{1}{8}$ is the square root of $\frac{1}{16}$, then $\frac{1}{8}$ times $\frac{1}{8}$ should equal $\frac{1}{16}$ which is not true.

Please return to page $\frac{94}{2}$ and try this question again.

Volume 8 Segment 4 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and 50	3	9	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	0	8	(Volume Number)
	66 and 68	0	4	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 245 - 248
You will now be asked a series of questions to draw your attention to the more important points.

In your reading you learned about multiplying two binomials of the form

$(a + b)$ and $(a - b)$, and the reverse operation; the factoring

of their product $a^2 - b^2$.

You will now be asked several questions to call your attention to the more important points.

Question 1
Perform the necessary operation to find the squares of the following numbers; 7 , 11 , 13. Select the letter indicating the correct set of squares.

(A) $\{14 , 22 , 26\}$ (C) $\{49 , 121 , 169\}$

(B) $\{49 , 121 , 143\}$ (D) $\{49 , 111 , 169\}$

Suppose we try a similar problem:

find the product of $(2ab - 1) (2ab + 1)$.

The only difference between the two binomials, is that one of them has a minus sign between its terms, while the other has a plus sign between its terms. The product of two such conjugate binomials is the square of the first term minus the square of the second term. Thus,

$$(2ab)^2 - 1^2 = 4a^2 b^2 - 1$$

is the answer. This rule is a great time saver; make a note of it in your notebook. You may write

$$(x + y) (x - y) = x^2 - y^2$$

Return to page 105/1 and try this question again.

$\frac{84}{1}$

Very good. You made the right choice.

Let us do the problem together.

The area of the shaded part

= the area of the semi-circle	- the area of the triangle.
= $\frac{1}{2}\pi$ times the radius squared	- $\frac{1}{2}$ base times height
= $\frac{1}{2}\pi r^2$	- $\frac{1}{2}$ b h
= $\frac{1}{2}\pi r^2$	- $\frac{1}{2}$ (2r) h
= $r\left(\frac{\pi r}{2} - h\right)$	

Please go on to question 18 below.

$\frac{84}{2}$

Question 18

Propose how you could simplify the computation of

$\frac{54 \times 98}{42 \times 63}$ by using factoring and find the value of the

above expression without performing the actual calculation. Select the letter which best describes the procedure you used.

(A) The best procedure is to use a slide rule.

(B) The best procedure is to multiply and divide as indicated.

(C) $\frac{54 \times 98}{42 \times 63} = \frac{9 \times 6 \times 98}{7 \times 6 \times 9 \times 7} = \frac{98}{49} = 2$

(D) $\frac{54 \times 98}{42 \times 63} = \frac{2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 2}{7 \cdot 3 \cdot 2 \cdot 7 \cdot 3 \cdot 3} = 2$

Very good. You made the right choice. The square of the first monomial is:

$$(8r^2t) (8r^2t) = 8 \cdot 8\, r^2 \cdot r^2 \cdot t \cdot t = 64r^4t^2$$

Similarly, the square of the second monomial is:

$$(-3uv^2)(-3uv^2) = (-3)(-3)\, u \cdot u \cdot v^2 \cdot v^2 = 9u^2v^4$$

Note that the product of two negative quantities is a positive quantity.

Please go on to question 3 below.

Question 3

Recognize that each of the numbers of the product (51) (49) can be written as binomials.

Select the letter which labels the correct answer that has the same two terms and differs only in the sign between the terms.

e.g. (a + b) (a - b)

(A) (51 + 0) (49 - 0)

(B) (50 + 1) (50 - 1)

(C) (49 + 2) (50 - 1)

(D) None of these.

86/1

A binomial can also be the common factor.

$x(x - 2) + 3(x - 2)$ has $(x - 2)$ as the common factor.

Just as in

$x^{a + 3a} = a(x + 3)$ this common factor can be removed and placed as a multiplier of the remaining polynomial. Therefore, the correct answer is

$$(x - 2)(x + 3)$$

Please go on to question 15 below.

86/2

Question 15

Apply the proper principle and write the following expression in completely factored form.

$$y^2 - y + yz - z$$

Select the letter which labels the correct answer.

(A) $yz(y - 1)$

(B) $y(y - 1) + z(y - 1)$

(C) $(y - 1)(y + z)$

(D) $y(y + z) - (y + z)$

$\frac{87}{1}$

Try multiplying 13 x 13 over again. We don't agree with the result you obtained.

Return to page $\frac{83}{1}$ and choose another letter.

$\frac{87}{2}$

You did not make the correct choice. You neglected an old rule.

When multiplying two quantities having the same base,

keep the base and add the exponents.

For example,

$$p \cdot p = p^{1+1} = p^2$$

Return to page $\frac{92}{2}$ and try the question again.

$\frac{88}{1}$

Very good. You made the right choice. Multiplication and division can often be simplified by factoring. Thus, in this question, we write each of the numbers as a product of primes,

$\frac{54 \times 98}{42 \times 63} = \frac{2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 2}{7 \cdot 3 \cdot 2 \cdot 7 \cdot 3 \cdot 3}$. Cancelling the like factors in the numerator and denominator, we are left with 2 in the numerator.

$\frac{88}{2}$

You have now completed this segment.

Hand in the PUNCH CARD.

Make sure that you have the following items in your notebook.

(1) Definition of the expression, factoring over a given set.

(2) Definition of the word prime number.

(3) Definition of the words common monomial factor, greatest common monomial factor.

You should now do question 9 - 12 of the homework assignment.

You showed some ingenuity in this choice.

$y^2 - y + yz - z$ can be arranged to

$y^2 + yz - y - z$

Now the first two terms $y^2 + yz$

can be factored into $y(y + z)$

and the second two terms $-y \quad -z$

can be factored into $-1(y + z)$

together they equal $y(y + z) - 1(y + z)$

But this expression can be factored further.

Return to page 86/2 and try this question again.

Suppose we make up a little formula:

a number = (its square root) (its square root)

With this formula you can easily check whether you have made the correct choice of a square root. To illustrate, let us check whether

$.08ab^2$ is the square root of $.04a^2b^4$

$.04a^2b^4$ should equal $(.08ab^2)(.08ab^2)$

Since

$(.08)(.08) = .0064$ we know that we have made a bad choice.

Please return to page 109/2 and choose another letter.

$\frac{90}{1}$

Try multiplying 11 x 11 over again. We have obtained a different answer than the one you did.

Please return to page $\frac{83}{1}$ and choose another letter.

$\frac{90}{2}$

You can check whether you have factored correctly by multiplying the two factors. Their product should equal the original expression. The question that you should answer is:

Does $(a - 2c)(a - 2c) = a^2 - 4c^2$?

Please return to page $\frac{93}{2}$ and try this question again.

$\frac{91}{1}$

The choice you made is not wrong, but adding or subtracting zero from a number is really equivalent to just writing the numbers as they were given in the question. There is a better choice and we are sure that you can find it.

Return to page $\frac{85}{2}$ and choose another letter.

$\frac{91}{2}$

To check whether you have factored correctly, you should multiply the two factors.

Thus, if your choice is correct,

$\left(y - \frac{1}{8} \right) \left(y + \frac{1}{8} \right)$ should equal $y^2 - \frac{1}{16}$, but it doesn't.

Please return to page $\frac{106}{2}$ and try this question again.

92
1

Very good. You made the correct choice. The product of

$(x + 3)(x - 3)$ can be obtained by direct multiplication; thus,

$$\begin{array}{r} x + 3 \\ \underline{x - 3} \\ x^2 + 3x \\ \underline{\quad - 3x - 9} \\ x^2 \quad - \quad 9 \end{array}$$

However, if we observe the two binomials whose product we see, we note that they are of the form

$(a + b)(a - b)$. These are called "conjugates". The product of such binomials is the square of the first term minus the square of the second term.

This formula should be memorized.

$$(a + b)(a - b) = a^2 - b^2$$

Please go on to question 5 below.

92
2

Question 5

Apply the proper principle to find the product

$(b - \frac{1}{2})(b + \frac{1}{2})$ and select the letter which labels the correct answer.

(A) $b - \frac{1}{4}$

(B) $b^2 - b - \frac{1}{4}$

(C) $b^2 - \frac{1}{4}$

(D) None of these.

Very good. You made the correct choice.

Since: $(.9x)(.9x) = .81x^2$,

therefore, $.9x$ is the square root of $.81x^2$.

Since: $(.2ab^2)(.2ab^2) = .04a^2b^4$,

therefore, $.2ab^2$ is the square root of $.04a^2b^4$.

Please go on to question 9 below.

Question 9

Apply the proper principle and write

$a^2 - 4c^2$ in factored form.

Select the letter which labels the correct answer.

(A) $(a - 4c) (a + 4c)$

(B) $(a - 2c) (a + 2c)$

(C) $(a - 2c) (a - 2c)$

(D) $(a^2 - 2c) (1 + 2c)$

$\frac{94}{1}$

Very good. You made the correct choice. The product of two such conjugate binomials can be written down at sight. These binomials are of the form

$$(a + b) (a - b)$$

Their product is $a^2 - b^2$, or the square of the first term minus the square of the second term.

Thus, we have

$$(xy - 1) (xy + 1) = (xy)^2 - 1^2$$

$$= x^2y^2 - 1$$

Please go on to question 7 below.

$\frac{94}{2}$

Question 7

Perform the necessary calculation to find the square roots of the numbers

$$0.01 \ , \frac{1}{16}$$

Select the letter which has the correct set of square roots next to it.

(A) .1 , $\frac{1}{8}$

(B) .0001 , $\frac{1}{4}$

(C) .1 , $\frac{1}{4}$

(D) .005 , $\frac{1}{32}$

This choice is correct.

Now proceed to question 2 which follows:

Question 2

Perform the necessary operation to find the squares of the two monomials,

$$8r^2t \quad \text{and} \quad -3uv^2$$

Select the letter next to the correct answers.

(A) $64r^4t^2$ / $9u^2v^4$

(B) $16r^4t^2$ / $-6u^2v^2$

(C) $64r^4t^2$ / $-9u^2v^4$

(D) $64r^4t$ / $9uv^4$

A good question to ask yourself is:

how much is $\frac{1}{2}$ times $\frac{1}{2}$.

The answer is not 1 , but $\frac{1}{4}$. Remember that $\frac{1}{2}$ of a number is not the same as the square root of that number.

Please return to page $\frac{110}{1}$ and try this question again.

$\frac{96}{1}$

You are confusing the square root of a number with $\frac{1}{2}$ of a number. They are not the same. You are looking for a number, which multiplied by itself will equal $\frac{1}{16}$. It may help you find such a number if you keep in mind that the square root of a proper fraction is the square root of the numerator divided by the square root of the denominator.

Return to page $\frac{106}{2}$ and try this question again.

$\frac{96}{2}$

If you read the question, you will see that the choice you made does not meet the requirements. Each of the numbers must be written as a binomial. The binomials must consist of the same numbers; the only difference being that one binomial has a plus sign, while the other has a minus sign. Here is an illustration of the type of binomials that we are looking for:

$$(77 - 5) (77 + 5).$$

Please return to page $\frac{85}{2}$ and try this question again.

You did not make the right choice.

What is the square root of $4c^2$?

Do you see your mistake?

Return to page 93/2 and try this question again.

When we multiply two decimal numbers each less than one, the product is a smaller number than the numbers we started with.

Thus, .2 times .2 = .04.

Clearly, .04 is smaller than .2. Thus, in order to obtain the number .09 as a result of multiplying two equal numbers, the equal numbers must be larger than .09.

Please return to page 107/1 and try this question again.

VIII

98/1

Go over your work and pay close attention to signs. We are sure that you will find your error. However, with binomials, one of which is the sum of two quantitites, the other is the difference of the same two quantities; (that is, when they are conjugate binomials) multiplication is not necessary. The product can be written at sight.

Please go over your reading assignment and study the rule for finding the product of such binomials.

Return to page 92/2 and try this question again.

98/2

Say this to yourself: if my answer is correct, then

(.0001) x (.0001) should equal .01

But, .0001 x .0001 = .00000001

Therefore, I am wrong.

Please return to page 94/2 and try this question again.

We do not agree. We would like to refresh your memory on a few matters.

(1) The square of a quantity means the quantity multiplied by itself. Thus, the square of $-3axy^2$ is

$$(-3axy^2)(-3axy^2) = 9a^2x^2y^4$$

(2) The square of a number is always a positive quantity.

(3) When two quantities having the same base are multiplied, the exponents are added.

Please return to page $\frac{95}{2}$ and try this question again.

Please check the choice you made by multiplying the two factors.

Does $(1 - x)(1 + 9x)$ equal $(1 - 9x^2)$?

Return to page $\frac{110}{1}$ and choose another letter.

$\frac{100}{1}$

We do not agree. One of the choices does represent the correct answer to this question.

Please return to page $\frac{99}{2}$ and choose another letter.

$\frac{100}{2}$

You should think of the binomial

$$(2x - 3y)$$ as a single term.

Let us go a step further and denote this binomial by a single letter; thus, let

$$u = 2x - 3y$$

Then, we have to factor

$$u^2 - 16z^2 .$$

This is the difference of two squares.

Please continue from here.

Return to page $\frac{111}{2}$ and try this question again.

You misunderstood.

Twice the sum is not the same as twice the product.

Please return to page 118/1 and choose another letter.

We not agree.

How do we factor an expression consisting of the difference of two squares? Let us do a problem.

Factor $9x^2 - 4y^2$

What is the square root of $9x^2$? Answer, $3x$.

What is the square root of $4y^2$? Answer, $2y$.

Now, one of the factors is the sum of these square roots, the other is the difference of these square roots;

thus, $(3x + 2y)(3x - 2y)$, is the answer.

Please return to page 93/2 and try this question again.

VIII

$\frac{102}{1}$

We do not agree.

You should remember that the two binomial factors of the difference of two squares have different signs.

Please return to page $\frac{106}{2}$ and try this question again.

$\frac{102}{2}$

We do not agree.

One of the choices does have the correct answer. Examine all the choices again.

Return to page $\frac{110}{1}$ and try this question again.

One of the choices does represent the correct answer. Examine each of the choices carefully.

Return to page $\frac{85}{2}$ and choose another letter.

The square root of a number is not the same as one-half that number.

You need the answer to this question:

what number multiplied by itself equals $\frac{1}{16}$?

Please return to page $\frac{94}{2}$ and try this question again.

$\frac{104}{1}$

Very good. You made the correct choice.

The square root of $.09r^2$ is $.3r$, and the square root of s^2 is s.

Hence, the factors are

$$(.3r + s) \text{ and } (.3r - s).$$

Please go on to question 13 below.

$\frac{104}{2}$

Question 13

Apply the proper principle to write

$$x^{2a} - y^{2a}$$ in factored form.

Select the letter which labels the correct statement.

(A) This is not the difference of two squares and cannot be factored.

(B) $(x^2 + y^a) (x^a - y^a)$

(C) $(x^a - y^a)^2$

(D) $x^a y^a (x^a - y^a)$

This choice is correct.

Now proceed to question 6 which follows:

Question 6

Apply the proper principle to find the product

$(xy - 1)(xy + 1)$ and select the letter next to the correct answer.

(A) $x^2 y^2$

(B) $x^2 y^2 - xy - 1$

(C) $x^2 y^2 - 2xy - 1$

(D) $x^2 y^2 - 1$

To find twice the product of two monomials, you first multiply them together and then double the result.

For example: to find twice the product of 3a and 4b. We first find the product

$(3a)(4b) = 12ab$, then we double the result

$2 (12ab) = 24ab.$

Please return to page 125/1 and try this question again.

106
1

A binomial in the form $a^2 - b^2$ can be factored as

$$(a + b)(a - b).$$

Therefore, $a^2 - 4c^2 = (a + 2c)(a - 2c)$

or $(a - 2c)(a + 2c)$

this can be verified by multiplying the two binomials.

$$\begin{array}{r} a - 2c \\ \underline{a + 2c} \\ a^2 - 2ac \\ \underline{+ 2ac - 4c^2} \\ a^2 \qquad - 4c^2 \end{array}$$

Please go on to question 10 below.

106
2

Question 10

Apply the proper principle to write $y^2 - \frac{1}{16}$ in factored form and select the letter which labels the correct answer.

Write the fractions in the form $\frac{1}{8}$.

(A) $(y - \frac{1}{8})(y + \frac{1}{8})$ (C) $(y - \frac{1}{4})(y + \frac{1}{4})$

(B) $(y - \frac{1}{32})(y + \frac{1}{32})$ (D) $(y - \frac{1}{4})(y - \frac{1}{4})$

This choice is correct.

Now proceed to question 12 which follows:

Question 12

Apply the proper principle to write

$.09r^2 - s^2$ in factored form and select the letter which labels the correct answer.

(A) $(.0003r + s)(.0003r - s)$

(B) $(.003r + s)(.003r - s)$

(C) $(.03r + s)(.03r - s)$

(D) $(.3r + s)(.3r - s)$

Twice the product of two quantities means that the product of these quantities is to be doubled.

For example, find twice the product of

$-4xy^2$ and $6x^2$

The product is $(-4xy^2)(6x^2) = -24x^3 y^2$. Notice the signs

The product doubled is $2 (-24x^3 y^2) = -48x^3 y^2$.

Let us try another example.

Find twice the product of

$(-3p^2 q)$ and $(-6pq^2)$.

The product is $(-3p^2 q)(-6pq^2) = 18p^3 a^3$. Notice the signs.

The product doubled is $2 (18p^3 q^3) = 36p^3 q^3$.

Return to page 112/2 and choose another letter.

108/1

This choice is correct.

Now proceed to question 4 which follows:

Question 4

Apply the proper principle to find the product

$(x + 3) (x - 3)$ and select the letter which labels the correct answer.

(A) $x^2 - 9$

(B) $x^2 + 6x - 9$

(C) $x^2 - 6x - 9$

(D) x^2

108/2

You seem to have mixed up different methods.

$(2x - 3y)^2$ is not the same as $(2x)^2 - (3y)^2$

Why don't you let $(2x - 3y) = a$ and $(16z) = b$.

Then the problem would become $a^2 - b^2$, which you know how to factor.

Please return to page 111/2 and try this question again.

Do not let the fact that the exponents are letters upset you.

You still would like to know the following:

Is x^{2a} the square of a quantity?

Is y^{2a} the square of a quantity?

If so, we have the difference of two squares, and it can be factored.

Please return to page 104/2 and try this question again.

This choice is correct.

Now proceed to question 8 which follows:

Question 8

Perform the necessary computation to find the square roots of each of the monomials:

$$.81x^2 \quad , \quad .04a^2 b^4$$

Select the letter which labels the correct answer.

(A) $.9x$, $.08ab^2$

(B) $.09x$, $.02ab^2$

(C) $.9x$, $.2a^2 b^2$

(D) $.9x$, $.2ab^2$

110
1

This choice is correct.

Now proceed to question 11 which follows:

Question 11

Apply the proper principle to write $1 - 9x^2$ in factored form and select the letter which labels the correct answer.

(A) $(1 + 3x)(1 - 3x)$

(B) $(\frac{1}{2} - 3x)(\frac{1}{2} + 3x)$

(C) $(1 - x)(1 + 9x)$

(D) None of these.

110
2

The square of a binomial is a trinomial. It is very important for you to remember this statement.

Keep in mind that there are three terms in the expansion of $(x + y)^2$.

$(a + b)^2 \neq a^2 + b^2$ check by letting: $a = 10$

$b = 2$

$(10 + 2)^2 \neq 100 + 4$

Please return to page 114
1 and try this question again.

Very good. The fact that we have literal exponents does not change the rule.

We note that $x^{2a} = (x^a)^2$, $y^{2a} = (y^a)^2$.

Thus, we have to factor the difference of two squares.

The factors are,

$(x^a + y^a)$ and $(x^a - y^a)$.

Please go on to question 14 below.

Question 14

Determine the proper principle and express

$(2x - 3y)^2 - 16z^2$ in factored form.

Choose the letter next to the correct answer.

(A) $(2x - 3y - 4z)^2$

(B) $(2x - 3y)(2x + 3y) - 16z^2$

(C) $(2x - 3y + 4z)(2x - 3y - 4z)$

(D) $4z(2x + 3y) - 4z(2x - 3y)$

112/1

We do not agree.

We are looking for twice the product, not merely the product.

Return to page 118/1 and choose another letter.

112/2

This choice is correct.

Now proceed to question 3 which follows:

Question 3

Perform the necessary calculation to find twice the product of

$$-3ab^2 \quad \text{and} \quad 6a.$$

Select the letter which labels the correct statement.

(A) $-9a^2b^2$

(B) $-18a^2b^2$

(C) $18a^2b^2$

(D) $-36a^2b^2$

Although $(x^a)^2$ equals x^{2a}

It is not true that

$$(x^a - y^a)^2 = x^{2a} - y^{2a}$$

The square of a binomial is a trinomial.

$$(a - b)^2 = a^2 - 2ab + b^2$$

Please return to page 104/2 and try this question again.

Part of the rule for squaring a binomial states " twice the product ". To make sure that your middle term is right, multiply the two terms of the binomial and then double this product.

Please return to page 134/1 and try this question again.

114
1

This choice is correct.

Now proceed to question 4 which follows:

Question 4

Apply the proper principle to find $(p + q)^2$ and select the letter which labels the correct statement.

(A) $p^2 + q^2$

(B) $2pq$

(C) $p^2 + 2pq + q^2$

(D) $p^2 q^2$

114
2

What is the square of 3r? Please note that the first term of the binomial has the letter " r " raised to the second power. Squaring this term will therefore result in increasing the power of " r ".

Please return to page 127/2 and try this question again.

Your answer is not wrong since you did factor the first term. However, you failed to realize that

$(2x - 3y)^2 - 16z^2$ is actually a case of " the difference of two squares ", and it can be factored the same way that $a^2 - b^2$ can be factored.

But here, of course, " a " stands for

$(2x - 3y)$.

With this hint you should be able to continue.

Please return to page 111/2 and try this question again.

We do not agree.

As you are finding the square of a binomial, you should be saying the following to yourself:

The square of the first term, plus twice the product of the first and second terms, plus the square of the second term. Thus, in expanding $(3rs + t)^2$ for example we notice that the square of the first term is

$(3rs)^2 = 9r^2 s^2$.

Twice the product of the first and second terms is

$2(3rs)(t) = 6rst$.

The square of the second term is

$(t)^2 = t^2$.

Please return to page 135/2 and try this question again.

116
1

The rule for finding the square of a binomial is to square the first term, add twice the product of the first and second terms; and add the square of the second term.

Examine your choice carefully. We are sure that you will discover the error that you made.

Please return to page 126/2 and try this question again.

116
2

Remember that part of the rule states, " twice the product " of the two terms of the binomial. What is twice the product of (5x) and (-3y)?

Please reconsider your choice.

Return to page 123/2 and choose another letter.

Check your answer by multiplication.

Note:

$x^a y^a (x^a) = x^{2a} y^a$ this should convince you that you made a poor choice.

Please return to page 104/2 and try this question again.

We do not agree.

Please remember that the square of a binomial is a trinomial.

You have a term missing.

Please return to page 134/1 and try this question again.

Volume 8 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	4 0	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 8	(Volume Number)
	66	and	68	0 5	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 248 - 249
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Perform the necessary calculation to find twice the product of 5 and 7.

Select the letter which labels the correct answer.

(A) 24 (C) 4

(B) 35 (D) 70

The square of the sum of two terms is equal to the square of the first term, plus twice the product of the first and the second, plus the square of the second term.

For example, $(x + 3)^2$ equals;

x^2 , the square of the first term,

$+ \quad 6x$, twice the product of the first and second terms,

$+ \quad 9$, the square of the last term.

This follows the rule:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Please return to page 114/1 and choose another letter.

Excellent. You answered a pretty difficult question correctly.

If we consider the binomial $(2x - 3y)$ as a single term, we have to factor the difference of two squares.

Now, the square root of

$$(2x - 3y)^2 \text{ is } 2x - 3y;$$

and the square root of

$$16z^2 \text{ is } 4z.$$

Hence, the factors are:

$$2x - 3y + 4z$$

and

$$2x - 3y - 4z.$$

You have now completed this segment. Make sure that you have the following items in your notebook:

(1) The product of the sum and difference of two numbers is the square of the first number minus the square of the second number. Thus,

$$(a + b) (a - b) = a^2 - b^2 .$$

(2) The difference of the squares of two numbers can be factored as illustrated by the following example:

$$a^2 - b^2 = (a + b) (a - b).$$

You should now do questions 13 - 16 of the homework assignment.

$\frac{120}{1}$

You found twice the difference.

In order to find twice the product, you first find the product and then double this product.

Please return to page $\frac{118}{1}$ and choose another letter.

$\frac{120}{2}$

The square of a binomial is <u>not</u> equal to the square of the first term plus the square of the second term. In other words.

$$(2a + 3b)^b \quad \text{does } \underline{\text{not}} \text{ equal } \quad (2a)^2 + (3b)^2 .$$

It is possible that you are saying to yourself, " why not "? Let us show why not. Suppose that we have $(3 + 7)^2$. Had we added the sum of the squares of the terms, we would have

$$3^2 + 7^2 = 9 + 49 = 58 .$$

But,

$$3 + 7 = 10 \quad \text{and} \quad 10^2 = 100.$$

Certainly, 100 does not equal 58.

Please return to page $\frac{126}{2}$ and try this question again.

Did you guess?

If you had read the assignment, you would have known that the square of a binomial is an expression of three terms.

This rule applies: $(a + b)^2 = a^2 + 2ab + b^2$

Return to page 114/1 and try this question again.

You did not make the right choice.

What is the square of $3k$?

Remember that the square of a number is the number multiplied by itself and not the number multiplied by 2.

Please return to page 134/1 and choose another letter.

$\frac{122}{1}$

What is twice the product of $3r^2$ and 7 ?

Please re-examine the middle term of your choice.

Return to page $\frac{127}{2}$ and choose another letter.

$\frac{122}{2}$

We do not agree.

We have no quarrel with the first and last terms of your answer, but the middle term is not correct. The middle term is found by taking twice the product of the two terms of the binomial.

What does

$$2 (a) (- \frac{b}{2})$$ equal to?

Do you see your mistake?

Please return to page $\frac{136}{2}$ and try this question again.

Although one method of squaring a binomial is to employ the rule which you should memorize:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Some students find it easier to say:

" the square of the first term
plus two times the product of the first and last term
plus the square of the last term ".

Your answer is correct.

Please go on to question 9 below.

Question 9

Apply the proper principle to expand $(5x - 3y)^2$

Select the letter next to the correct answer.

(A) $25x^2 - 15xy + 9y^2$

(B) $15x^2 - 30xy + 6y^2$

(C) $25x^2 - 30xy + 9y^2$

(D) $25x^2 + 9y^2$

$\frac{124}{1}$

We do not agree. You forgot an old rule.

When a power is raised to a power, the exponents are multiplied.

i.e. $\boxed{(x^a)^b = x^{ab}}$

For example, $(3x^2y)^2 = 3^2 (x^2)^2 (y^1)^2 = 9x^4y^2$.

Please return to page $\frac{133}{1}$ and try this question again.

$\frac{124}{2}$

Let us give you a small hint.

Suppose you re-write

$(2x - 3y + 1)^2$ as

$[(2x - 3y) + 1]^2$

In other words, think of $(2x - 3y)$ as a single term and apply the rule for squaring a binomial.

Please return to page $\frac{132}{2}$ and try this question again.

This choice is correct.

Now proceed to question 2 which follows.

Question 2

Perform the necessary calculation to find twice the product of 3m and 2n . Select the letter which is next to the correct answer.

(A) 6 mn

(B) 12 mn

(C) 24 mn

(D) 6 mn + 4 n

This choice is correct.

Now proceed to question 12 which follows.

Question 12

Apply the proper principle to expand $(x^a + y^b)^2$

Select the letter which labels the correct statement.

(A) $x^{a^2} + 2x^a y^a + y^{a^2}$

(B) $x^{2a} + y^{2b}$

(C) $x^{2a} + 2x^a y^b + y^{2b}$

(D) $x^{a^2} + y^{a^2}$

Very good. You made the correct choice.

$(p + q)^2$ is the same as $(a + b)^2$

i.e. $(p + q)^2 = p^2 + 2pq + q^2$.

This rule should be memorized.

Please go on to question 5 below.

Question 5

Apply the proper principle to expand $(3x + 1)^2$. Select the letter next to the correct answer.

(A) $9x^2 + 3x + 1$

(B) $9x^2 + 6x + 1$

(C) $9x^2 + 1$

(D) $6x + 2$

You should memorize the rule.

$$\boxed{(a - b)^2 = a^2 - 2ab + b^2}$$

Then $(3k - 2)^2$ would have $a - 3k$

$b = 2$

and substituting in the rule

$$(3k - 2)^2 = (3k)^2 - 2(3k)(2) + (2)^2$$
$$= 9k^2 - + 4$$

Your answer choice was correct.

Please go on to question 7 below.

Question 7

Apply the proper principle to expand $(3r^2 - 7)^2$. Select the letter which labels the correct statement.

(A) $9r^2 - 42r + 49$

(B) $6r^4 - 42r + 49$

(C) $9r^4 - 49$

(D) $9r^4 - 42r^2 + 49$

128
1

The square of a number is the number multiplied by itself. Thus, the square of $5x$ is $(5x)(5x)$, and the square of $3y$ is $(3y)(3y)$. Look over the choice you made. We are sure that you will discover your error.

Return to page 123/2 and choose another letter.

128
2

By just looking at your choice, you know that it is not correct.

All that you did was to square each term.

You must apply the rule

$$(x - y)^2 = x^2 - 2xy + y^2 .$$

Please return to page 136/1 and try another letter.

$\frac{129}{1}$

The square of a binomial is a trinomial. Since your choice contains only two terms, please reconsider.

Return to page $\frac{133}{1}$ and try this question again.

$\frac{129}{2}$

Allow us to refresh your memory concerning the rule for raising a power to a power. When a power is raised to a power, the exponents are multiplied.

For example: $(x^3)^2 = x^6$. Similarly,

$(r^a)^2 = r^{2a}$, and not r^{a^2}.

Please return to page $\frac{125}{2}$ and try this question again.

130/1

The square of a binomial does <u>not</u> equal the square of the first term plus the square of the second term. Since the square of a binomial is a trinomial, you have a term missing. Please find it.

Return to page 127/2 and try this question again.

130/2

What is the square of 12? That's easy you say. The square of 12 is 144. Now, $12 = 10 + 2$, so that $(10 + 2)^2$ should also equal 144. If we take the sum of the squares of the two terms of the above binomial we get, $10^2 + 2^2 = 100 + 4 = 104$. This answer is 40 less than the correct answer. How do we get this number 40 ? We must add twice the product of the two terms of the binomial; thus, $2(10)(2) = 40$. What is the moral of the above illustration? Don't forget the middles term when you square a binomial.

Please return to page 123/2 and try this question again.

The square of a number is not the same as twice that number. The expression

$$(3x + 1)^2$$

is not the same as

$$2(3x + 1).$$

Return to page 126/2 and try this question again.

The rule for squaring a binomial says in part, "twice the product".

We are sure that you made your choice hastily.

Please return to page 133/1 and choose another letter.

132
1

Very good. You made the correct choice. We have to expand

$$(x^a + y^b)^2 .$$

The square of x^a is $(x^a)^2 = x^{2a}$.

Twice the product of x^a and y^b is $2x^a y^b$.

The square of y^b is $(y^b)^2 = y^{2b}$.

Please go on to question 13 below.

132
2

Question 13

Relate to the proper principle to simplify the expression,

$$(2x - 3y + 1)^2 .$$

Select the letter next to the correct statement.

(A) $4x^2 + 9y^2 + 1$

(B) $4x^2 - 12xy + 9y^2 + 1$

(C) $4x^2 - 12xy + 9y^2 + 4x - 6y + 1$

(D) None of these.

This choice is correct.

Now proceed to question 10 which follows.

Question 10

Apply the proper principle to expand $(x^2 - 3y^2)^2$.

Select the letter which labels the correct statement.

(A) $x^2 - 6x^2y^2 + 9y^2$

(B) $x^4 - 6x^2y^2 + 9y^4$

(C) $x^4 - 9y^4$

(D) $x^4 - 3x^2y^2 + 9y^4$

We do not agree. One of the letters has the correct answer to this question. Do you remember the rule for squaring a binomial? Square the first term; add twice the product of the first and second terms; add the square of the second term.

Please return to page 136/2 and choose another letter.

134
1

This choice is correct.

Now proceed to question 6 which follows.

Question 6

Apply the proper principle to expand $(3k - 2)^2$

Select the letter which labels the correct answer.

(A) $9k^2 - 12k + 4$

(B) $9k^2 - 6k + 4$

(C) $9k^2 - 4$

(D) $6k^2 - 12k + 4$

134
2

You handled the squaring of x^a and y^b properly, but you forgot the rule for squaring a binomial.

Please return to page 125/2 and try this question again.

The expression $(3r^2 - 7)^2$ can be considered a form of $(a - b)^2$ which equals $a^2 - 2ab + b^2$ when expanded.

By careful substitution when $3r^2 = a$, and $b = 7$,

we get $(3r^2)^2 - 2 (3r^2) (7) + (7)^2$

or $9r^4 - 4^2r + 49$

which is your answer choice.

Please go on to question 8 below.

Question 8

Apply the proper principle to expand $(2ax + b)^2$.

Select the letter next to the correct answer.

(A) $4a^2x^2 + b^2$

(B) $4a^2x^2 + 4axb + b^2$

(C) $4a^2x^2 + 2axb + b^2$

(D) $4abx$

136
1

To expand $(x^2 - 3y^2)^2$, we refer to the basic formula

$$(a - b)^2 = a^2 - 2ab + b^2$$

Therefore, $$(x^2 - 3y^2)^2 = (x^2)^2 - 2(x^2) (3y^2) + (3y^2)^2$$

$$= x^4 - 6x^2y^2 + 9y^4$$

Your choice was correct.

Please go on to question 11 below.

136
2

Question 11

Apply the proper principle to expand $(a - \frac{b}{2})^2$

Select the letter which labels the correct statement.

(A) $a^2 - \frac{ab}{2} + \frac{b^2}{4}$

(B) $a^2 - \frac{b^2}{4}$

(C) $a^2 - ab + \frac{b^2}{4}$

(D) None of these.

Excellent. You did a pretty difficult problem correctly. Let us do this problem together. We have

$$(2x - 3y + 1)^2$$

We will consider (2x - 3y) as a single term and rewrite the above as,

$$\left[(2x - 3y + 1) \right]^2$$

This can be considered as the square of a binomial; thus,

$$(2x - 3y)^2 + 2(2x - 3y) + 1$$

Expanding again, we get,

$$4x^2 - 12xy + 9y^2 + 4x - 6y + 1$$

You have now finished this segment.

PUNCH CARD: Before you hand in your punch card to the instructor, check to see that you have entered the required information; particularly your course student number, and this course number and the sequence number of the segment just completed which is 40.

NOTEBOOK: You should have entered in your notebook the following formulas and definitions:

(1) $(a + b)^2 = a^2 + 2ab + b^2$

(2) $(a - b)^2 = a^2 - 2ab + b^2$

(3) $(a + b + c)^2 = \left[(a + b) + (c) \right]^2$

$= (a + b)^2 + 2(a + b)(c) + c^2$

HOMEWORK ASSIGNMENT: You are now able to complete the homework assignment. Hand in your homework before you take the volume test. You are now advised to review all of your notes and to re-read the reading assignment in order to prepare yourself for the volume test.

1/16/70

Programmed Mathematics Continuum — ERRATA SHEET — Attach to back Cover

ALGEBRA - LEVEL ONE — VOLUME 8

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected. Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE		MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
$\frac{44}{2}$		$\frac{52}{2}$	$\frac{53}{2}$	
$\frac{100}{1}$		$\frac{99}{2}$	$\frac{92}{2}$	
$\frac{48}{1}$	Line 6	—	$\frac{\cdot}{\cdot}$	
$\frac{22}{2}$	Line 2	—	$\frac{\cdot}{\cdot}$	
$\frac{36}{2}$	Line 2	—	$\frac{\cdot}{\cdot}$	
$\frac{53}{2}$	Line 2	—	$\frac{\cdot}{\cdot}$	
$\frac{40}{2}$	Line 2	—	$\frac{\cdot}{\cdot}$	
$\frac{86}{2}$	Line 4	x^{a+3a}	$xa + 3a$	